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Lumpy investment and variable capacity utilization: firm-level and macroeconomic implications

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Abstract

The macroeconomic implications of firms' lumpy investment behavior are subject to ongoing research. Lumpy investment results from fixed capital adjustment costs which give firms an incentive to reduce the frequency of capital adjustments. However, previous studies have underestimated the lumpiness. Their assumption of constant capital utilization reduces firms' incentives to undertake large investments as it prevents reserve capacity building. This paper shows that if capacity utilization is allowed to vary, firms optimally undertake larger investments and leave parts of the new capital stock idle for some periods, thereby reducing the frequency of investment activities. Using a dynamic stochastic general equilibrium model with fixed capital adjustment costs, heterogeneous firms, variable utilization, and aggregate technology shocks, I numerically compute firms' optimal decisions on investment, utilization and labor demand. Compared to the constant utilization model, the findings reveal magnified investment lumpiness: Firms adjust capital less frequently, but invest more when they adjust. However, this appears to be of minor macroeconomic relevance: Moments and impulse responses of macroeconomic quantities change in a similar way when variable utilization is introduced in a lumpy or in a frictionless model. New empirical evidence based on firm-level panel data confirms some of the theoretical findings.

JEL Classification: E22, E32, D92

Keywords: lumpy investment, adjustment costs, reserve capacity, utilization, business cycles

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1 Introduction

Analyzing the impact of aggregate shocks on aggregate endogenous variables is a major objective of many macroeconomic models. Because aggregate outcomes result from the interaction of individual agents' choices, macroeconomic dynamics may essentially depend on microeconomic frictions and the heterogeneity of economic agents. This has become an important issue in the analysis of investment dynamics because firm-level capital adjustments are characterized by substantial frictions and heterogeneity. There are periods of inaction which are occasionally interrupted by large alterations. Such *lumpy*, i.e., infrequent and large, adjustments have been established as a prevalent feature of investment at the establishment level (e.g., Doms and Dunne, 1998, Cooper et al., 1999, Cooper and Haltiwanger, 2006, and Gourio and Kashyap, 2007). This lumpy investment behavior has been widely interpreted as evidence for non-convexities in capital adjustment costs. For example, if capital adjustment is subject to fixed costs, firms are only willing to invest if their capital stock deviates sufficiently from the desired level. Supporting evidence is provided, for example, by Caballero et al. (1995), Caballero and Engel (1999) and Cooper et al. (1999).

These non-convex adjustment costs and the resulting lumpy investment behavior at the micro level do not necessarily affect macroeconomic dynamics and the impulse responses of aggregate variables. On the one hand, aggregation smooths lumpy investment activities to the extent that they are not synchronized. On the other hand, general equilibrium price movements may additionally smooth investment spikes that survive aggregation. Disregarding this second argument, partial equilibrium models suggest that lumpy microeconomic investment is potentially important for aggregate investment (examples include Caballero et al., 1995, Caballero and Engel, 1999, Cooper et al., 1999, and Cooper and Haltiwanger, 2006). In contrast, several general equilibrium models find negligible aggregate effects of lumpy investment (e.g., Thomas, 2002, and Khan and Thomas, 2003, 2008). However, this *irrelevance result* does not generically follow from general equilibrium. To what extent the effect of fixed costs disappear in general equilibrium is a quantitative question that depends on the details of the model calibration (Gourio and Kashyap, 2007). For example, Bachmann et al. (2013) show that lumpy investment can explain a stylized fact of US macroeconomic data, namely the conditional heteroscedasticity of the aggregate investment to capital ratio. This ratio is substantially more responsive to shocks in booms than in recessions. The results of Gourio and Kashyap (2007) are also indicative of lumpiness affecting the impulse responses of aggregate investment. Bachmann and Ma (2012) and Bachmann and Bayer (2014) provide additional examples in which microeconomic lumpiness matters for macroeconomic analysis.

The existing lumpy investment models posit that firms always fully utilize their capital. However, this assumption is delicate in the presence of fixed capital adjustment costs. Firm-level investment and capacity utilization decisions are inherently interrelated. The assumption of constant utilization reduces the firms' incentives for large investments, resulting in an underestimation of investment lumpiness. In particular, the incentives to build up reserve capacity are attenuated. The following example illustrates this issue. In an environment with long-run technological progress, the firms' desired capital services increase over time. Absent fixed investment costs, firms can simply adjust their capital stock to the optimal level in each period regardless of whether utilization is variable. In the presence of fixed costs, however, firms have an incentive to reduce the number of capital adjustments. In particular, it may be optimal to invest up to $\tilde{k} > k^*$, where k^* denotes next period's optimal capital stock absent fixed costs, and to omit investment and the associated fixed costs in the subsequent period. This is the core idea behind lumpy investment: Because of fixed capital adjustment costs, firms have an incentive to invest more than currently needed in order to relieve themselves of the need to readjust too soon. As a consequence, firms prefer rare and large over frequent and small adjustments. However, this key incentive for large investments is attenuated in the existing lumpy investment models by requiring firms to fully utilize their capital. Intuitively, if firms must use their capital stock $\tilde{k}$ in next period's production, they have an incentive to select $\tilde{k}$ close to k^*. If, in contrast, this assumption of full utilization is dropped, then firms can easily invest up to a capital stock that is considerably larger than k^*. At the same time, they can achieve optimal capital services k^* by not fully utilizing their large capital stock. To sum up, if forward-looking firms carry out an investment project, they choose the size of the investment big enough such that, once the project is completed, there is some excess or reserve capacity for future growth and no immediate need to undertake the next investment project.

This paper relaxes the delicate assumption of constant capacity utilization, thereby allowing for amplified microeconomic investment lumpiness due to reserve capacity building. It contributes to the lumpy investment literature by providing an analysis of (i) how variability in capacity utilization alters firms' investment decisions and (ii) how, if at all, macroeconomic variables are affected by the enhanced microeconomic lumpiness. For this analysis, I use a real business cycle (RBC) model with heterogeneous firms, fixed capital adjustment costs and variable utilization. The economy is subject to aggregate productivity shocks. Apart from variability of utilization, the model is closely related to the setup considered by Khan and Thomas (2003). Firms differ in their capital stock and in the current draw of fixed investment costs. They jointly decide on labor demand, utilization, whether to invest at all, and next period's capital stock (if they invest). The utilization choice is conceptually very different for investing and non-investing firms: It is an intratemporal choice for the former and an intertemporal decision for the latter. The household side, in contrast, is kept simple: Households decide on consumption, labor supply and the amount of shares to buy.

The extension of the lumpy investment model by variable utilization is motivated by at least three additional reasons besides the aforementioned proper consideration of investments in reserve capacity. First, this paper presents new firm-level evidence highlighting the importance of variable utilization for firms' investment decisions. Using panel data, I show that firm-level capacity utilization has a direct impact on individual firms' probability of investment. Moreover, there are significant interaction effects with GDP growth. For example, lagged GDP growth only has a positive impact on the investment probability if the utilization rate is sufficiently high. An aggregation exercise demonstrates that forcing firm-level utilization rates to be constant alters aggregate investment properties across the business cycle. Second, because preferences for smooth consumption restrict the macroeconomic relevance of lumpy investment in general equilibrium, a potentially greater impact of lumpiness might only emerge in models that attenuate the tight link between consumption and investment dynamics. Variable capacity utilization may provide a way to relax this tight link because, in a standard RBC model, it leads to smoother consumption and more volatile investment.[1] Third, variable capacity utilization concedes a limited intertemporal choice to firms refraining from paying the fixed adjustment costs. In contrast, many previous studies have assumed that these firms cannot influence the evolution of their capital stock, an assumption that has been criticized.[2] In this paper, non-investing firms affect their depreciation rate and, consequently,

[1] An alternative strategy is used by Bachmann and Ma (2012), who consider a model with capital goods heterogeneity to relax the tight link between consumption and investment dynamics and to enhance households' ability to smooth consumption. Indeed, they find that fixed capital adjustment costs are of macroeconomic relevance. Their results indicate that the response of fixed capital investment to aggregate productivity shocks differs in both magnitude and persistence depending on the presence of fixed capital adjustment costs.

[2] For example, Khan and Thomas (2008, p.396) point out that: *"... as is the convention throughout the literature, there was a stark assumption that nonconvex adjustment costs applied to all capital adjustments irrespective of their size"*. Khan and Thomas (2008) address this issue by allowing for low levels of capital adjustment without incurring fixed costs. They assume that the range of investment rates exempt from such

their future capital stock by changing current utilization. In contrast to Khan and Thomas (2008), their influence on next period's capital is naturally asymmetric: Non-investing firms cannot increase capital, but reduce its depreciation by lowering utilization.

The results of this paper shed light on individual firms' investment, utilization and labor decisions in an environment with fixed capital adjustment costs. The findings reveal that variable utilization magnifies lumpiness: Fewer firms invest on average, but the adjusting firms choose a considerably higher capital stock. Moreover, firms with a lot of capital (i.e., firms that have recently invested) choose a utilization rate substantially below the maximum feasible rate. Thus, this paper provides evidence for reserve capacity building which causes enhanced investment lumpiness: Investing firms optimally choose a capital stock that is too large in the short run and partially lies idle. Additional evidence for amplified lumpiness is provided by an analysis of the investment rate distribution, which reveals that if utilization is variable, firm-level investments relative to their capital stock are larger on average, more volatile, more asymmetric and, in particular, feature substantially higher kurtosis.

The variability of utilization also affects firms' optimal decisions on labor demand and utilization. Additionally, it induces some cyclical differences: First, the target capital level of investing firms increases more strongly with productivity when utilization is variable. Second, the probability of investment fluctuates to a greater extent across the cycle. The latter finding is confirmed by empirical evidence presented in this paper.

Regarding differences in the behavior of investing and non-investing firms, the results indicate that non-investing firms usually choose a substantially lower utilization rate to save capital for future years. Also, their labor demand is slightly lower in many cases.

While this paper's results establish amplified investment lumpiness at the firm level owing to variable utilization, the macroeconomic consequences of the lumpiness are less clear-cut. Simulation results suggest that, despite the larger investment of those firms that adjust their capital, the means of most macroeconomic variables differ only slightly across the constant and variable utilization model. Regarding the volatility of macroeconomic aggregates, variable utilization has a more pronounced impact. The variables characterizing investment lumpiness (i.e., the target capital of adjusting firms and the fraction of such firms) are more volatile in this paper's model. The same holds for output, investment, labor, capital, and the investment ratio, whereas consumption becomes less volatile. However, this pattern does not specifically

costs is symmetric around zero.

pertain to the lumpy investment models. Note that allowing for variable capacity utilization also has macroeconomic effects in a model without fixed capital adjustment costs (henceforth labeled *frictionless model*). Hence, the important question is whether the enhanced lumpiness caused by variable utilization has an impact beyond the one that can be expected from introducing variable utilization in a frictionless model. The results do not provide evidence in favor of that. The standard deviation of macroeconomic aggregates appears to change in a similar way when variable utilization is introduced in a frictionless or in a lumpy investment model.

A similar conclusion holds for the impulse responses to aggregate technology shocks: The responses of this paper's model differ considerably from those of the lumpy investment model with constant utilization,[3] but are comparable to those of a frictionless model with variable utilization. In particular, the initial responses of output, investment, consumption and employment hardly depend on the existence of fixed capital adjustment costs. However, notable differences pertain to the impulse response functions of capital, utilization and the target capital of investing firms. In this paper's model, the response of aggregate capital is larger and more persistent while aggregate utilization initially increases to a smaller extent compared to the frictionless model with variable utilization. The initial response of investing firms' target capital is even massively larger. Since the response of aggregate investment is similar though, this means that the additional investment in the aftermath of a positive productivity shock is undertaken by few firms which invest a lot rather than all firms that invest a small amount.

The remainder of the paper is structured as follows. Section 2 presents new empirical evidence based on firm-level panel data highlighting the importance of capacity utilization for investment both at the micro and macro level. In section 3, I describe the building blocks of the theoretical model, the competitive equilibrium, the specification and the calibration of the model. The method for the numerical model solution is explained in section 4. The results are discussed in section 5. It presents optimal firm-level decisions on investment, utilization and labor in an environment with variable utilization and fixed capital adjustment costs, and the macroeconomic consequences thereof. Finally, section 6 summarizes and concludes.

[3]Inter alia, the amplified lumpiness is apparent in the considerably larger responses of the fraction of adjusting firms and the target capital of these firms.

2 Empirical Analysis

This section investigates the relevance of variable capacity utilization for both firm-level and aggregate investment. The analysis is based on quarterly firm-level panel data from the KOF Swiss Economic Institute. I use answers to the KOF business tendency survey of the manufacturing industry, a poll of Swiss industrial companies from a wide range of industrial sectors that participate voluntarily.[4]

The main outcome of interest are two binary indicators about investment. In the survey, firms are asked whether their technical production capacity was (i) increased, (ii) left unchanged, or (iii) decreased in the preceding three months. I use this categorical variable to construct two binary indicators: first, an investment dummy equal to one if production capacity was increased and zero if it was left unchanged or decreased and, second, a disinvestment dummy equal to one if production capacity was decreased and zero if it was left unchanged or increased.

The key explanatory variables are lagged firm-level capacity utilization and lagged real GDP growth. Firms are asked to state their average utilization rate of production capacity in the preceding three months. The variable is categorical, ranging from 50% utilization up to 110% in steps of 5 percentage points. Given this rather fine grid of possible answers, I treat the variable as continuous. Real GDP growth rates are obtained from the State Secretariat of Economic Affairs SECO.[5]

Table 1 shows basic descriptive statistics for my sample. It includes 1824 firms from 2004 Q1 to 2014 Q3 (43 quarters). Not all firms are observed over the entire period. Overall, there are 36328 firm-quarter observations for the investment indicators and 32369 for capacity utilization.

Table 1: Descriptive statistics

	Mean	SD	obs
Investment (yes/no)	13.8%		36328
Disinvestment (yes/no)	4.2%		36328
Capacity utilization	82.3%	14.2	32369
Real GDP growth rate	0.5%	0.6	43

Notes: Mean, standard deviation (SD) and number of observations (obs)

[4] For additional information such as sample questionnaires, see http://www.kof.ethz.ch/en/surveys/business-tendency-surveys/manufactoring/.

[5] Data are available at http://www.seco.admin.ch/themen/00374/00456/04878/index.html?lang=en.

To investigate how capacity utilization affects investment, I specify a linear panel data model for the investment decision of firm i in period t with firm-specific effects and interaction terms as

$$d_{it} = \sum_{j=1}^{4} \beta_j GDP_{t-j} + \sum_{j=1}^{4} \gamma_j u_{it-j} + \sum_{j=1}^{4} \delta_j GDP_{t-j} u_{it-j} + c_i + \varepsilon_{it}, \tag{1}$$

where d_{it} is the decision on either investment or disinvestment in period t, u_{it-j} denotes capacity utilization, c_i is a firm-specific unobserved effect that may be correlated with the regressors u_{it-j}, and ε_{it} is a time-varying error. Moreover, time dummies are included, but not shown for readability.[6]

Different strategies can be used to remove the unobserved effect c_i from equation (1). I use the within transformation, which subtracts the average over time of (1) from the model equation:

$$\begin{aligned} d_{it} - d_{i\cdot} = & \sum_{j=1}^{4} \beta_j (GDP_{t-j} - GDP_{\cdot}) + \sum_{j=1}^{4} \gamma_j (u_{it-j} - u_{i\cdot}) \\ & + \sum_{j=1}^{4} \delta_j (GDP_{t-j} u_{it-j} - GDP_{\cdot} u_{i\cdot}) + \varepsilon_{it} - \varepsilon_{i\cdot}, \end{aligned} \tag{2}$$

where a dot in the subscript indicates time-averages, e.g., $d_{i\cdot} = T^{-1} \sum_{t=1}^{T} d_{it}$. The pooled OLS estimator of the demeaned equation (2) is called the fixed effects (FE) estimator. Under the assumption of strict exogeneity,

$$\mathbb{E}(\varepsilon_{it} | GDP_1, \ldots, GDP_T, u_{i1}, \ldots, u_{iT}, c_i, \text{time dummies}) = 0, \quad t = 1, \ldots, T, \tag{3}$$

the FE estimator is consistent and unbiased. Given that GDP is an aggregate variable whereas ε_{it} is firm-specific, strict exogeneity is likely to hold. However, the firm-specific error ε_{it} may possibly affect future capacity utilization, rendering u in (2) potentially endogenous.

To solve the potential endogeneity problem associated with FE estimation of (1), I rely

[6] Because the dependent variables are binary, (1) represents a linear probability model. Logit or probit models are often used to analyze binary choices, but since these models do not easily generalize to the combination of fixed effects and dynamics (as considered in model (4)), I confine the analysis to linear probability models. This type of model has well-known limitations. It implies heteroscedastic errors, which I address by computing heteroscedasticity-robust standard errors. Moreover, estimated probabilities from this model are not restricted to the unit interval. However, because I am interested in estimating mean effects, this issue should not substantially affect the results.

on lagged instruments from the panel. In particular, the first-differenced error $\varepsilon_{it} - \varepsilon_{it-1}$ may be correlated with $u_{it-1} - u_{it-2}$, but should be unrelated to u_{it-2} and further lags, which can therefore be used as instruments. This procedure relies on ε_{it} being serially uncorrelated. To ensure this, I include lagged dependent variables in equation (1) and specify the following linear dynamic panel data model:

$$d_{it} = \sum_{j=1}^{p} \alpha_j d_{it-j} + \sum_{j=1}^{4} \beta_j GDP_{t-j} + \sum_{j=1}^{4} \gamma_j u_{it-j} + \sum_{j=1}^{4} \delta_j GDP_{t-j} u_{it-j} + c_i + \varepsilon_{it}, \tag{4}$$

which additionally includes time dummies. The number of dependent variable lags p is chosen such that ε_{it} is serially uncorrelated. $p = 1$ is sufficient in the model with the investment dummy as dependent variable whereas $p = 2$ is needed in the model for the disinvestment dummy.

I estimate (4) using the system GMM estimator proposed by Blundell and Bond (1998), who build on the work of Arellano and Bover (1995). As outlined in Arellano and Bond (1991), estimation of (4) may proceed by first-differencing the equation to remove the fixed effect and using d_{it-2}, u_{it-2} and further lags to instrument Δd_{it-1} and Δu_{it-1} on the right-hand side. System GMM augments this so-called difference GMM by additionally using lagged differences as instruments for the level equation.[7] Thus, system GMM estimates simultaneously in levels and differences with different instruments for the two equations. The moment conditions are given as follows:

$$\mathbb{E}[d_{it-l}\Delta\varepsilon_{it}] = 0, \qquad \mathbb{E}[u_{it-l}\Delta\varepsilon_{it}] = 0, \quad \text{for } t \geq 3, l \geq 2, \tag{5}$$

$$\mathbb{E}[\Delta d_{it-1}(c_i + \varepsilon_{it})] = 0, \qquad \mathbb{E}[\Delta u_{it-k}(c_i + \varepsilon_{it})] = 0, \quad \text{for } t \geq 3, k = 1, \ldots, 4. \tag{6}$$

I perform the estimation using the Stata command *xtabond2* of Roodman (2009b).

The number of instruments is quadratic in the time dimension of the panel because the number of applicable lags in (5) increases with T. As discussed in Roodman (2009a), too many instruments (*instrument proliferation*) is a common problem in connection with system GMM. It is problematic because many instruments can cause overfitting of endogenous variables and imprecise estimation of the optimal weighting matrix. Because my sample consists of 43 quarters of observations, the empirical analysis based on the system GMM estimator is prone

[7] This is valid under the assumption that changes in any instrumenting variable are uncorrelated with c_i.

to the problems associated with instrument proliferation. Therefore, I restrict the number of instruments.

I apply two different ways to reduce the number of instruments. The first consists in reducing the moment conditions (5) by considering only a few instead of all available lags. I estimate various model specifications, the largest model using up to the sixth lag as instruments. The second way consists in collapsing the instrument set (see Roodman, 2009a). The moment conditions (5) and (6) are replaced by

$$\mathbb{E}[d_{it-l}\Delta\varepsilon_{it}] = 0, \qquad \mathbb{E}[u_{it-l}\Delta\varepsilon_{it}] = 0, \quad \text{for } l \geq 2, \tag{7}$$

$$\mathbb{E}[\Delta d_{it-1}(c_i + \varepsilon_{it})] = 0, \qquad \mathbb{E}[\Delta u_{it-k}(c_i + \varepsilon_{it})] = 0, \quad \text{for } k = 1, \ldots, 4, \tag{8}$$

in which case the instrument count is linear in the time dimension of the panel. Just as (5) and (6), the moment conditions (7) and (8) embody the belief of orthogonality between differenced errors and lagged levels of d and u, and between errors in levels and lagged differences of d and u. However, in contrast to the moment conditions (5) and (6), the estimator only minimizes $\sum d_{it-l}\Delta\varepsilon_{it}$ and $\sum u_{it-l}\Delta\varepsilon_{it}$ for each l rather than for each l and t separately. Similarly, $\sum \Delta d_{it-1}(c_i + \varepsilon_{it})$ and $\sum \Delta u_{it-k}(c_i + \varepsilon_{it})$ are only minimized in total, not separately for each t. Thus, less information is conveyed by a collapsed instrument set. On the other hand, no lags are actually dropped, which is a potential advantage over the approach of reducing the number of instruments by capping the number of lags used as instruments.

Applying the system GMM estimator involves many specification choices. Therefore, it is important to check the robustness of the results. I use various specifications: Anderson and Hsiao (1982) difference and level estimators, which use only either the most recent lagged difference or lagged level as instruments for the first difference of equation (4), the difference GMM estimator proposed by Arellano and Bond (1991), and the system GMM estimator discussed above with either a restricted number of lags used as instruments or with a collapsed instrument set. Despite some quantitative differences, the results are qualitatively robust to different specifications of the GMM estimator. The aggregate findings (presented in figures 1 and 2 below) and the resulting conclusions are even more robust. First stage regressions of level variables on first-differences and of first-differenced variables on lagged levels clearly confirm the strength of the instruments used in the different model specifications.

Table 2 presents the estimated coefficients of the FE and system GMM model with the investment indicator as dependent variable. Table 3 shows the corresponding results for the disinvestment dummy. Columns (1) and (3) contain the estimates for the models without interaction terms. The estimated coefficients are of the expected sign: Both recent GDP growth and utilization have a positive impact on the probability of investment and a negative impact on the probability of disinvestment.

Columns (2) and (4) of tables 2 and 3 present the estimates for the models including interaction terms. The findings show a significant impact of capacity utilization on the probability of investment. Moreover, some of the interaction terms are significant as well, indicating that the impact of lagged GDP growth on the probability of investment or disinvestment hinges on the firm-level utilization rate. For example, according to the results in column (2) of table 2, the effect of an increase in GDP growth by one percentage point on the probability of investment two quarters later is given by:

$$\frac{\partial d_{it}}{\partial GDP_{t-2}} = -0.0548 + 0.0009 u_{it-2}. \tag{9}$$

Thus, the impact of GDP_{t-2} on the investment probability is zero at a utilization rate of 60%. For firms with a utilization rate of 100%, however, an increase in GDP growth by one percentage point increases the probability of investment two quarters later by 3.6 percentage points. Thus, lagged GDP growth only has a positive effect on the investment probability if the utilization rate is sufficiently high.

Overall, the findings in tables 2 and 3 show that firm-level investment decisions across the business cycle crucially depend on capacity utilization. Not only does the investment decision directly depend on utilization, but the rate of capacity utilization also influences how firm-level investment responds to aggregate GDP growth.

In the remainder of this section, I assess whether the relevance of variable capacity utilization for investment decisions at the firm-level carries over to aggregate quantities. To this end, I compare different aggregate investment and disinvestment time series, which are constructed as follows: The regression results from column (4) in table 2 are used to predict the probability of investment, which is then aggregated across firms using employment-based weights. Moreover, an alternative probability of investment is predicted with capacity utilization restricted to be fixed at the firm-specific mean. This alternative probability is also aggregated across firms. The resulting employment-weighted fractions of investing firms are

Table 2: Regression results for investment dummy

	FE		GMM	
	(1)	(2)	(3)	(4)
GDP_{t-1}	0.0214***	-0.0375	0.0187***	-0.0328
	(0.0073)	(0.0259)	(0.0072)	(0.0483)
GDP_{t-2}	0.0215***	-0.0548**	0.0207**	-0.0498
	(0.0076)	(0.0228)	(0.0080)	(0.0313)
GDP_{t-3}	0.0116	0.0162	0.0020	0.0149
	(0.0078)	(0.0249)	(0.0086)	(0.0345)
GDP_{t-4}	0.0075	-0.0405	0.0195***	-0.0576
	(0.0163)	(0.0282)	(0.0075)	(0.0414)
u_{it-1}	0.0026***	0.0023***	0.0022***	0.0019***
	(0.0003)	(0.0004)	(0.0004)	(0.0005)
u_{it-2}	0.0006*	0.0001	0.0004	0.0000
	(0.0003)	(0.0003)	(0.0004)	(0.0004)
u_{it-3}	0.0007**	0.0008**	0.0009**	0.0010**
	(0.0003)	(0.0004)	(0.0004)	(0.0004)
u_{it-4}	0.0001	-0.0002	0.0002	-0.0003
	(0.0003)	(0.0003)	(0.0004)	(0.0005)
$GDP_{t-1}u_{it-1}$		0.0007**		0.0006
		(0.0003)		(0.0006)
$GDP_{t-2}u_{it-2}$		0.0009***		0.0008**
		(0.0003)		(0.0004)
$GDP_{t-3}u_{it-3}$		0.0000		-0.0001
		(0.0003)		(0.0004)
$GDP_{t-4}u_{it-4}$		0.0006**		0.0009*
		(0.0003)		(0.0005)
d_{it-1}			0.2021***	0.2005***
			(0.0162)	(0.0161)
Number of observations	18063	18063	17055	17055
Number of firms	1151	1151	1124	1124
Arellano-Bond test for AR(2): p-value			0.354	0.378
Arellano-Bond test for AR(3): p-value			0.935	0.941
Hansen test: p-value			0.783	0.772

Source: KOF Business Tendency Survey Manufacturing Industry 2004-2014, own calculations.
Notes: All models include year dummies. Cluster-robust standard errors in parantheses. (3) and (4) are estimated by the Blundell and Bond (1998) system GMM estimator with lagged dependent variable and lagged utilization (and interactions) instrumented. The instrument set is collapsed (cf. Roodman, 2009a), which results in an instrument count of 228 and 371 for (3) and (4). The third and second last line show the p-values for the Arellano and Bond (1991) autocorrelation tests of order 2 and 3. The last line presents the p-value of the over-identification test proposed by Hansen (1982).
* $p < 0.1$.
** $p < 0.05$
*** $p < 0.01$

Table 3: Regression results for disinvestment dummy

	FE		GMM	
	(1)	(2)	(3)	(4)
GDP_{t-1}	-0.0099**	-0.0138	-0.0093**	0.0368
	(0.0038)	(0.0198)	(0.0039)	(0.0303)
GDP_{t-2}	-0.0224***	-0.0248	-0.0212***	-0.0088
	(0.0070)	(0.0230)	(0.0072)	(0.0277)
GDP_{t-3}	0.0010	0.0276	0.0139*	0.0626**
	(0.0064)	(0.0204)	(0.0078)	(0.0247)
GDP_{t-4}	-0.0011	-0.0532**	-0.0045	-0.0003
	(0.0100)	(0.0212)	(0.0069)	(0.0280)
u_{it-1}	-0.0013***	-0.0014***	-0.0011***	-0.0008**
	(0.0003)	(0.0003)	(0.0003)	(0.0003)
u_{it-2}	-0.0003	-0.0003	-0.0004	-0.0003
	(0.0002)	(0.0003)	(0.0003)	(0.0003)
u_{it-3}	0.0003*	0.0005**	0.0000	0.0003
	(0.0002)	(0.0003)	(0.0002)	(0.0003)
u_{it-4}	0.0004*	0.0001	0.0000	0.0001
	(0.0002)	(0.0003)	(0.0003)	(0.0003)
$GDP_{t-1}u_{it-1}$		0.0000		-0.0005
		(0.0002)		(0.0003)
$GDP_{t-2}u_{it-2}$		0.0000		-0.0002
		(0.0003)		(0.0003)
$GDP_{t-3}u_{it-3}$		-0.0003		-0.0006**
		(0.0002)		(0.0003)
$GDP_{t-4}u_{it-4}$		0.0006***		0.0000
		(0.0002)		(0.0003)
d_{it-1}			0.2637***	0.2625***
			(0.0236)	(0.0236)
d_{it-2}			0.0658***	0.0643***
			(0.0202)	(0.0202)
Number of observations	18063	18063	16973	16973
Number of firms	1151	1151	1124	1124
Arellano-Bond test for AR(2): p-value			0.346	0.314
Arellano-Bond test for AR(3): p-value			0.437	0.412
Hansen test: p-value			0.550	0.860

Source: KOF Business Tendency Survey Manufacturing Industry 2004-2014, own calculations.

Notes: All models include year dummies. Cluster-robust standard errors in parantheses. (3) and (4) are estimated by the Blundell and Bond (1998) system GMM estimator with lagged dependent variable and lagged utilization (and interactions) instrumented. The instrument set is collapsed (cf. Roodman, 2009a), which results in an instrument count of 265 and 408 for (3) and (4). The third and second last line show the p-values for the Arellano and Bond (1991) autocorrelation tests of order 2 and 3. The last line presents the p-value of the over-identification test proposed by Hansen (1982).

* $p < 0.1$.

** $p < 0.05$

*** $p < 0.01$

plotted in figure 1.[8] The figure reveals that restricting utilization to be constant results in a different fraction of investing firms. The difference is cyclical: The restricted series underpredicts investment activity in booms and overpredicts it in periods of low GDP growth. Figure 2 shows an analogous analysis for the fraction of firms with negative investment. The prediction of this fraction appears to be too low in recessions and too high in booms if utilization is fixed. Figure 3 plots the difference in the predicted fraction of investing firms if utilization is allowed to vary or kept constant. Figure 4 depicts the corresponding difference in the predicted fraction of firms with negative investment. The differences are significant at the one percent level.[9]

The aggregate results for the fraction of investing firms do hardly depend on the underlying econometric model (FE or system GMM): The difference between the predicted fraction of investing firms when utilization is either variable or forced to be constant is almost identical for both models (cf. figures 3 and 17). Clearly, this difference is cyclical. This also holds for the predicted fraction of firms with negative investment, although the difference between the FE and the system GMM results is more pronounced (cf. figures 4 and 18). The results depicted in figures 1 to 4 are robust to the use of alternative instrument sets in the system GMM estimation.

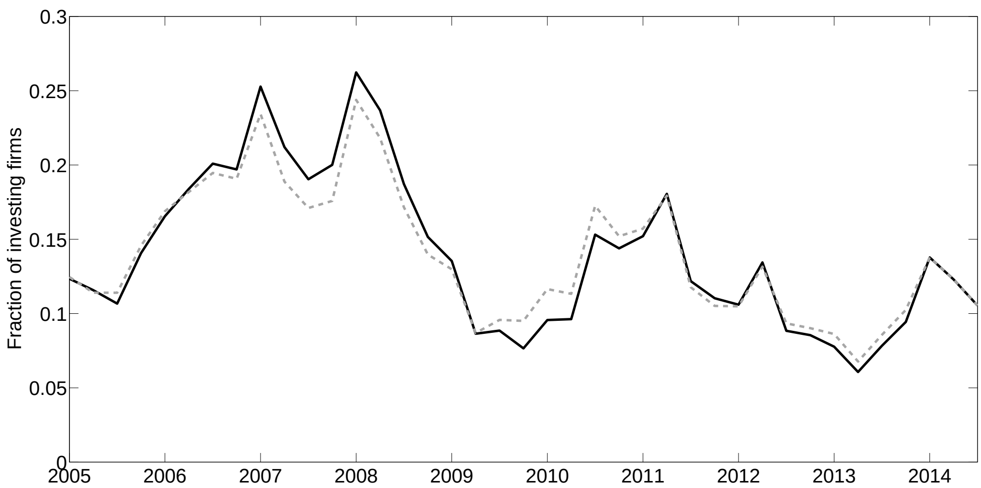


Figure 1: The fraction of investing firms when capacity utilization is fixed (gray dashed line) or variable (black solid line). The estimation is based on column (4) in table 2.

[8]Appendix A contains figures analogous to 1 to 4, but based on the FE (column (2) in the tables 2 and 3) instead of the system GMM results.

[9]This holds for the differences plotted in figures 3, 4, 17 and 18. Confidence intervals are computed using the delta method. They are very narrow and therefore not depicted.

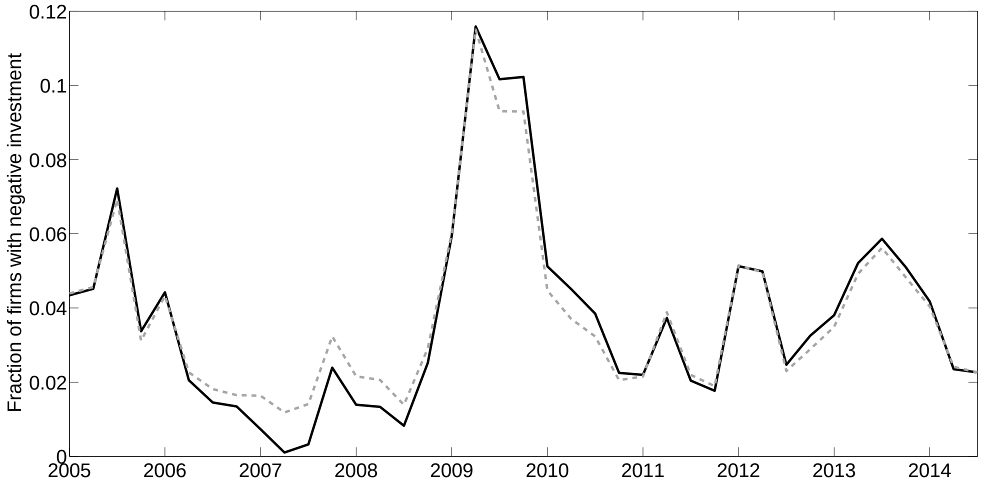


Figure 2: The fraction of firms with negative investment when capacity utilization is fixed (gray dashed line) or variable (black solid line). The estimation is based on column (4) in table 3.

Figure 3: Difference in the predicted fraction of investing firms when utilization is variable or fixed. The difference is based on the GMM model (column (4) in table 2).

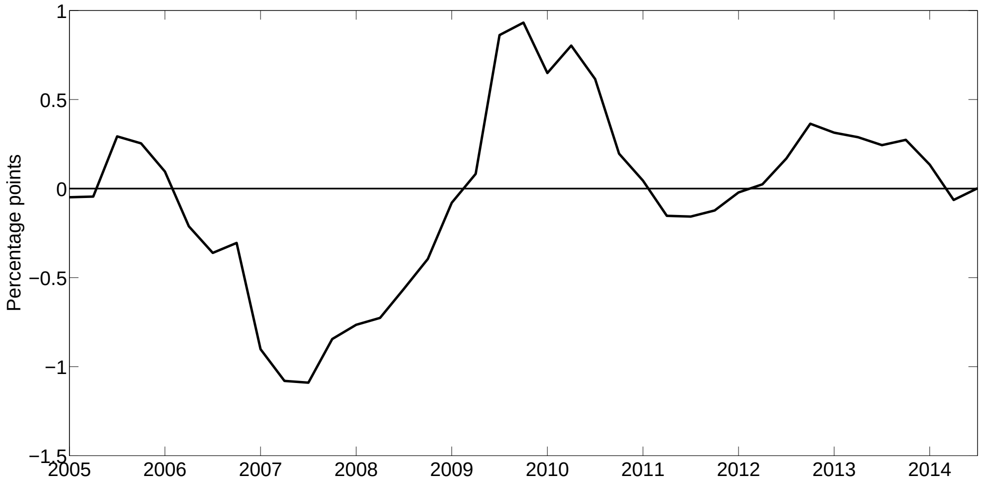


Figure 4: Difference in the predicted fraction of firms with negative investment when utilization is variable or fixed. The difference is based on the GMM model (column (4) in table 3).

Overall, the empirical analysis suggests that capacity utilization is a relevant variable in lumpy investment models. Compared to a fixed capacity utilization, high utilization rates in booms cause a larger fraction of firms to invest and a lower fraction to disinvest, which (depending on the size of the capital adjustment) may have macroeconomic effects. Analogously, low utilization rates in recessions cause a smaller fraction of firms to invest and a larger fraction to disinvest. In the light of this empirical evidence, capacity utilization appears to be an important ingredient in models assessing the relevance of lumpy firm-level investment for macroeconomic investment.

3 Theoretical model

I consider an RBC model with variable capacity utilization, heterogeneous firms and capital adjustment costs. Within any period, these capital adjustment costs are fixed at the level of a production unit, but may differ across units. As a consequence, some production units will undertake capital adjustments while others will not, resulting in firm heterogeneity with respect to the capital stock.

The model features long-run growth due to technological progress. Let γ denote the gross growth rate of the economy along the balanced growth path caused by trend growth in productivity. In the following exposition, I consider the detrended model, i.e., all variables measured in units of output are deflated.

Overall, the framework is closely related to the setups considered by Khan and Thomas (2003, 2008), Bachmann et al. (2013) and Bachmann and Bayer (2014). The main departure from these papers is to allow for variable capacity utilization, thereby introducing an intensive margin along which capital input to production can be adjusted. The following sections describe the model in detail.

3.1 Firms

The economy consists of a continuum of production units, henceforth labeled firms.[10] Because I do not model firms' entry and exit decisions, the mass of firms can be normalized to one. At any date, each firm is characterized by its predetermined capital stock k and its fixed cost of investment $\kappa \in [0, B]$. This fixed cost is denominated in hours of labor. Thus, a firm deciding to adjust its capital stock incurs costs of κw, where w denotes the real wage. In each period, κ is drawn from a time-invariant distribution G, which is continuous and has support $[0, B]$. The distribution G is common across firms. The draws from G are independent both over time and across firms.

There is a single commodity in the economy that can be consumed or invested. Each firm produces this commodity using capital services and labor as inputs. Capital services are given by the product of the capital stock k and capacity utilization u. In each period, k is predetermined while u can be adjusted up to an upper bound $\bar{u}$.[11] Labor n can be adjusted without frictions. Each firm's production is described by the function:

$$y = zF(ku, n), \tag{10}$$

which satisfies the following properties:[12]

$$F_1 > 0, F_2 > 0, F_{11} < 0, F_{22} < 0, F_{12} \geq 0.$$

z denotes exogenous aggregate productivity which is common across all firms. As in Khan

[10] Different interpretations of the size of production units are possible. For example, Cooper and Haltiwanger (2006) and Khan and Thomas (2008) assume that the production units in the model correspond to plants, while Bloom (2009) sets the number of productive units per firm at 250 for his simulation.

[11] Specifically, utilization can amount to 100% at most. However, I calibrate the model such that the steady state utilization rate equals one in the model without capital adjustment costs. Because of this normalization, $\bar{u} > 1$ and $u \in (1, \bar{u}]$ indicates utilization rates which exceed steady state utilization in the frictionless model.

[12] Functions with subscript numbers represent derivatives of the function with respect to the indicated arguments.

and Thomas (2003, 2008), I assume that z follows a Markov chain with finite states $z \in \{z_1, \dots, z_{N_z}\}$, $\Pr(z' = z_j | z = z_i) = \pi_{ji}$ and $\sum_{j=1}^{N_z} \pi_{ji} = 1$ for each $i = 1, \dots, N_z$.[13]

The aggregate state of the economy is described by (z, μ) where μ denotes the distribution of capital across firms. This distribution of firm-level capital evolves according to a law of motion Γ which depends on the aggregate state of the economy: $\mu' = \Gamma(z, \mu)$. Γ will be described in more detail below.

Each firm's capital stock evolves according to:

$$\gamma k' = (1 - \delta(u))k + i. \tag{11}$$

γ denotes the steady state gross growth rate of capital. Depreciation is an increasing, convex function of utilization, which is a standard way to model the costs associated with higher utilization (cf. King and Rebelo, 2000). i denotes investment. $i \neq 0$ requires paying the fixed costs of κw.

The firms maximize the expected sum of discounted profits by making a discrete decision about investment and by choosing labor n, utilization u, and next period's capital stock k'. These decisions are interrelated. The optimal choices of labor and utilization may differ for investing and non-investing firms. This difference is apparent for capacity utilization: For firms that choose to invest, the decision on u is intratemporal and independent of the choice on k'. In contrast, the choice on u is an intertemporal decision for non-investing firms because u determines k' through equation (11). Thus, unlike in Khan and Thomas (2003), Bachmann et al. (2013) or Bachmann and Bayer (2014), non-investing firms can make an intertemporal decision, albeit the choice set is limited to the attainable values of next period's capital stock, $k' \in [(1 - \delta(\bar{u}))k/\gamma, (1 - \delta(0))k/\gamma]$.

Let $v^1(k, \kappa; z, \mu)$ represent the expected discounted value of a firm with individual state variables k and κ when the aggregate state of the economy is (z, μ). The expected value prior to the adjustment cost draw κ amounts to

$$v^0(k; z, \mu) = \int_0^B v^1(k, \kappa; z, \mu) G(d\kappa). \tag{12}$$

Let $d_{z_j}(z_i, \mu)$ denote the discount factor that firms apply to their future value if aggregate

[13] Throughout this paper's model description, time subscripts are omitted and next period's variables are denoted with a prime.

productivity is z_i in the current and z_j in the next period. With this notation, the optimization problem of a firm can be stated as dynamic programming problem:

$$
\begin{aligned}
&v^1(k,\kappa;z,\mu) = \\
&\max\Bigg\{\sup_{u,n}\left[zF(ku,n) - wn + (1-\delta(u))k\right] - \kappa w \\
&\qquad + \sup_{k'}\left[-\gamma k' + \mathbb{E}[d_{z'}(z,\mu)v^0(k';z',\mu')]\right]; \qquad (13)\\
&\qquad \sup_{u}\left[\sup_{n}(zF(ku,n) - wn) + \mathbb{E}\left[d_{z'}(z,\mu)v^0\left(\frac{(1-\delta(u))k}{\gamma};z',\mu'\right)\right]\right]\Bigg\}.
\end{aligned}
$$

The outer maximization represents a binary choice on investing. Let $I^*(k,\kappa;z,\mu) \in \{0,1\}$ denote the optimal binary choice. The last line in (13) describes the optimization problem of a firm that decides not to adjust its capital stock. Such a firm faces an intratemporal choice on labor n and an intertemporal choice on utilization u, which affects current production $zF(ku,n)$ and next period's capital $k' = \frac{(1-\delta(u))k}{\gamma}$. The associated policy functions are denoted by $n^f_N(k;z,\mu)$ and $u_N(k;z,\mu)$, respectively.[14] Lines two and three in (13) represent the optimization problem of a firm that decides to adjust its capital stock, thereby incurring fixed costs of κw. The problem is formulated as if investing firms sell their capital stock remaining after depreciation and purchase $\gamma k'$. This formulation is equivalent to merely subtracting investment from current profits, but it is more convenient because it reveals that the decisions on k' on the one hand as well as n and u on the other hand are separable. The optimal choice of u and n is intratemporal and does not depend on the choice on next period's capital. The associated policy functions are denoted $u_I(k;z,\mu)$ and $n^f_I(k;z,\mu)$. Similarly, the optimal k' is independent of the choices on u and n. In fact, (13) reveals that the optimal choice on k' is also independent of k and κ. Therefore, it does not depend on any firm-specific variable, but only on the aggregate state of the economy. As a result, any firm choosing to undertake capital adjustment will choose the same capital stock, denoted by $k^*(z,\mu)$.

There is an equivalent, but simpler representation of the dynamic programming problem in (13) (see Khan and Thomas, 2003, 2008). This representation incorporates optimality conditions from the households and competitive equilibrium conditions. Therefore, I will proceed by presenting the household sector (section 3.2), characterizing the competitive equilibrium

[14]The superscript f is used to differentiate labor choices of firms from those of households. The subscript N indicates non-investing firms. In contrast, subscript I will indicate investing firms.

(section 3.3) and then analyzing the optimality conditions from the simplified dynamic problem (section 3.4). In section 3.5, I finally present the specification and calibration of the functions and parameters of the model.

3.2 Households

The economy is populated by a continuum of identical households.[15] Their wealth is held as (one-period) shares in firms, which are denoted using the measure λ. Thus, the households own the portfolio of firms in the economy. In each period, households choose consumption C and supply labor N. Moreover, they decide on the amount of new shares $\lambda'(k')$ to buy of firms which begin the next period with a capital stock of k'. Economically, this is a portfolio choice problem with infinitely many assets indexed by k'. The optimization problem is given as follows:

$$
\begin{aligned}
W(\lambda; z, \mu) &= \sup_{C,N,\lambda'} U(C, 1-N) + \beta \mathbb{E}\left[W(\lambda'; z', \mu')\right] \\
\text{s.t.} \quad C + \int_{\mathcal{K}} \rho(k')\lambda'(dk') &\leq wN + \int_{\mathcal{K}} v^0(k)\lambda(dk),
\end{aligned}
\tag{14}
$$

with discount factor $\beta < 1$ and where $U(C, 1-N)$ satisfies the following properties:

$$U_1 > 0, U_2 > 0, U_{11} < 0, U_{22} \leq 0.$$

w denotes the real wage, $\rho(k')$ the price of new shares, and capital is defined on $\mathcal{K} \subseteq \mathbb{R}_+$.

Let p denote the Lagrange multiplier associated with the constraint in the optimization problem (14). The first order conditions for households' optimal choices are standard and given as follows:

$$U_1(C, 1-N) = p, \tag{15}$$

$$U_2(C, 1-N) = pw, \tag{16}$$

$$p\rho(k') = \beta \mathbb{E}\left[v^{0\,\prime}(k')p'\right], \quad \text{for each } k' \in \mathcal{K}. \tag{17}$$

Let $C(\lambda; z, \mu)$ and $N^h(\lambda; z, \mu)$ denote households' choice of consumption and labor, respectively. Moreover, let $\Lambda^h(k', \lambda; z, \mu)$ denote the amount of shares that households buy of firms

[15]Being alike, these households act like a single, representative household which takes prices as given.

which begin the next period with capital k'.

3.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium is a set of functions

$$\left(w, d, \rho, v^0, v^1, k^*, n_I^f, n_N^f, u_I, u_N, I^*, W, C, N^h, \Lambda^h, \Gamma\right)$$

that solve the firms' problem (13), the households' problem (14) and clear all markets. In particular, the set of functions satisfies:

(i) *Firm optimality:* Taking w, d and Γ as given, $v^0(k; z, \mu)$ and $v^1(k, \kappa; z, \mu)$ satisfy (12) and (13) with corresponding policy functions $I^*(k, \kappa; z, \mu)$, $n_N^f(k; z, \mu)$, $u_N(k; z, \mu)$, $n_I^f(k; z, \mu)$, $u_I(k; z, \mu)$ and $k^*(z, \mu)$.

(ii) *Household optimality:* Taking w, ρ, v^0 and Γ as given, $W(\lambda; z, \mu)$ satisfies (14) with corresponding policy functions $C(\lambda; z, \mu)$, $N^h(\lambda; z, \mu)$ and $\Lambda^h(k', \lambda; z, \mu)$.

(iii) *Asset market clearing:* The quantity of "capital k' shares" bought corresponds to the mass of firms with capital k', i.e., $\Lambda^h(k', \mu; z, \mu) = \mu'(k')$ for each $k' \in \mathcal{K}$.[16]

(iv) *Labor market clearing:* Labor supply is equal to labor demand for production and fixed costs of capital adjustment, which is denoted in units of labor:

$$\begin{aligned} N^h(\mu; z, \mu) = \int_{\mathcal{K}} \int_0^B & \Big(n_N^f(k; z, \mu)(1 - I^*(k, \kappa; z, \mu)) \\ & + n_I^f(k; z, \mu) I^*(k, \kappa; z, \mu) \Big) G(d\kappa)\mu(dk) \\ + \int_{\mathcal{K}} \int_0^B & I^*(k, \kappa; z, \mu)\kappa G(d\kappa)\mu(dk). \end{aligned}$$

[16] The household sector can be summarized by a representative household. Therefore, the measure λ of shares will coincide with the distribution of firm-level capital μ in equilibrium. Thus, I replace λ by μ in the household's policy functions.

(v) *Goods market clearing:* Consumption and gross investment add up to output:

$$C(\mu; z, \mu) + \int_{\mathcal{K}} \int_0^B \left(\gamma k^*(z,\mu) - (1 - \delta(u_I(k; z, \mu)))\, k\right) I^*(k, \kappa; z, \mu) G(d\kappa)\mu(dk)$$
$$= \int_{\mathcal{K}} \int_0^B \Bigg(zF\left(ku_N(k; z, \mu), n_N^f(k; z, \mu)\right)(1 - I^*(k, \kappa; z, \mu))$$
$$+ zF\left(ku_I(k; z, \mu), n_I^f(k; z, \mu)\right) I^*(k, \kappa; z, \mu)\Bigg) G(d\kappa)\mu(dk).$$

(vi) *Model consistent dynamics:* The evolution of the firm-level capital distribution, $\mu' = \Gamma(z, \mu)$, is induced by the exogenous process for z as well as optimal firm choices affecting next period's capital, namely $k^*(z, \mu)$, $I^*(k, \kappa; z, \mu)$ and $u_N(k; z, \mu)$:

$$\mu'(k') = \Gamma(z, \mu) = \int_{\left\{(k,\kappa)|k' = \frac{(1-\delta(u_N(k;z,\mu)))k}{\gamma} \text{ and } I^*(k,\kappa;z,\mu)=0\right\}} G(d\kappa)\mu(dk)$$
$$+ \int_{\{(k,\kappa)|k'=k^*(z,\mu) \text{ and } I^*(k,\kappa;z,\mu)=1\}} G(d\kappa)\mu(dk).$$

The first integral in the above expression integrates over all combinations of k and κ for which firms optimally choose not to invest and which result in next period's capital being k'. The second integral is only relevant for $k' = k^*(z, \mu)$ and zero for $k' \neq k^*(z, \mu)$. It integrates over all combinations of k and κ for which firms optimally choose to invest. This mass of the current capital distribution shifts to $k^*(z, \mu)$ in the next period.

3.4 Simplified Dynamic Problem

The firms' optimization problem can be simplified using equilibrium implications of household utility maximization (see Khan and Thomas, 2003, 2008). The arising problem is equivalent to the model presented in the previous sections, but consists of a single Bellman equation instead of (13) and (14), which simplifies equilibrium calculation.

The first step of this reformulation consists in finding equilibrium real wages and intertemporal prices using household optimality and general equilibrium conditions. An expression for the equilibrium real wage is obtained by combining the first order conditions (15) and (16):

$$w(z, \mu) = \frac{U_2(C, 1 - N)}{U_1(C, 1 - N)}, \tag{18}$$

where C and N denote the market-clearing values of consumption and labor. Combining asset market clearing with the household budget constraint leads to the following equation:

$$C + \int_{\mathcal{K}} \rho(k')\mu'(dk') = wN + \int_{\mathcal{K}} v^0(k)\mu(dk) = wN + \int_{\mathcal{K}} \int_0^B v^1(k,\kappa)G(d\kappa)\mu(dk).$$

Next, I plug in for v^1 and use labor and goods market clearing as well as model consistent dynamics. This results in the relation

$$\int_{\mathcal{K}} \rho(k')\mu'(dk') = \int_{\mathcal{K}} \mathbb{E}\left[d_{z'} v^{0\,\prime}(k')\right] \mu'(dk').$$

Finally, solving the household first order condition (17) for $\rho(k')$ and substituting it in the above expression yields:

$$\int_{\mathcal{K}} \mathbb{E}\left[\frac{\beta p'}{p} v^{0\,\prime}(k')\right] \mu'(dk') = \int_{\mathcal{K}} \mathbb{E}\left[d_{z'} v^{0\,\prime}(k')\right] \mu'(dk').$$

Thus, the discount factor applied by firms is equal to

$$d_{z'}(z,\mu) = \frac{\beta p'(z',\mu')}{p(z,\mu)}, \tag{19}$$

where $p(z,\mu)$ equals marginal utility of consumption.

Following Khan and Thomas (2003, 2008), I use the discount factor implied by equation (19) to write the firms' optimization problem in terms of household utils instead of physical output units:[17]

$$\begin{aligned}
&V^1(k,\kappa;z,\mu) = \\
&\max\Bigg\{\sup_{u,n}\left[zF(ku,n) - wn + (1-\delta(u))k\right]p - \kappa wp \\
&\qquad + \sup_{k'}\left[-\gamma k'p + \beta\mathbb{E}[V^0(k';z',\mu')]\right]; \\
&\qquad \sup_{u}\left[\sup_{n}(zF(ku,n) - wn)p + \beta\mathbb{E}\left[V^0\left(\frac{(1-\delta(u))k}{\gamma};z',\mu'\right)\right]\right]\Bigg\},
\end{aligned} \tag{20}$$

where the price firms use to value their current output is given by $p = U_1(C, 1-N)$, real

[17] For notational clarity, let V^0 and V^1 denote the value functions associated with the problem in terms of household utils while v^0 and v^1 denote the corresponding functions of the problem in terms of output.

wages are determined by $w = \frac{U_2(C,1-N)}{U_1(C,1-N)}$, and

$$V^0(k;z,\mu) = \int_0^B V^1(k,\kappa;z,\mu)G(d\kappa). \tag{21}$$

Optimal labor and capacity utilization choices of an investing firm (n_I^f and u_I) depend on the aggregate state (z,μ) and the individual capital stock k. They are characterized by the following first order conditions:

$$w = zF_2(ku_I, n_I^f), \tag{22}$$

$$\delta_u(u_I) = zF_1(ku_I, n_I^f). \tag{23}$$

The optimal capital choice k^* of investing firms is independent of current capital k and capital adjustment costs κ and satisfies:

$$\gamma p = \beta \mathbb{E}\left[V_1^0(k^*;z',\mu')\right]. \tag{24}$$

Thus, all investing firms choose the same capital stock for the next period. The target capital k^* depends on the aggregate state (z,μ).

The first order conditions for non-investing firms are given by

$$w = zF_2(ku_N, n_N^f), \tag{25}$$

$$pzF_1(ku_N, n_N^f) = \frac{1}{\gamma}\delta_u(u_N)\beta\mathbb{E}\left[V_1^0\left(\frac{(1-\delta(u_N))k}{\gamma};z',\mu'\right)\right]. \tag{26}$$

These conditions determine optimal employment and capacity utilization (n_N^f and u_N) as a function of k and (z,μ).

Finally, the binary investment decision is based on the comparison of the expected value of adjusting capital and incurring the fixed costs on the one hand with the expected value of foregoing adjustment on the other hand. The firm invests if

$$V_I^* - \kappa w p \geq V_N^*, \tag{27}$$

where

$$V_N^*(k;z,\mu) \equiv (zF(ku_N,n_N^f) - wn_N^f)p + \beta\mathbb{E}\left[V^0\left(\frac{(1-\delta(u_N))k}{\gamma};z',\mu'\right)\right], \tag{28}$$

$$V_I^*(k;z,\mu) \equiv (zF(ku_I,n_I^f) - wn_I^f + (1-\delta(u_I))k)p - \gamma k^* p + \beta\mathbb{E}\left[V^0(k^*;z',\mu')\right]. \tag{29}$$

It follows that a firm invests if and only if its fixed cost draw κ is below a certain threshold, namely if

$$\kappa \leq \bar{\kappa}(k;z,\mu) = \min\left\{\frac{V_I^*(k;z,\mu) - V_N^*(k;z,\mu)}{w(z,\mu)p(z,\mu)};B\right\}. \tag{30}$$

This amounts to a *reservation price* or *reservation cost* strategy, i.e., there is a maximum cost of $\bar{\kappa}(k;z,\mu)w(z,\mu)p(z,\mu)$ which a firm with capital k is willing to pay for the possibility to adjust its capital.

Given the investment decision described in (30), the cross-sectional distribution of firm-level capital evolves according to the following law of motion:

$$\begin{aligned}\mu'(k') = \Gamma(z,\mu) = &\int_{\left\{k \mid k'=\frac{(1-\delta(u_N(k;z,\mu)))k}{\gamma}\right\}} \left(1 - G\left(\bar{\kappa}\left(k;z,\mu\right)\right)\right)\mu(dk) \\ &+ \int_{\mathcal{K}} \mathbf{1}\{k' = k^*(z,\mu)\} G\left(\bar{\kappa}\left(k;z,\mu\right)\right)\mu(dk).\end{aligned} \tag{31}$$

$\mathbf{1}\{k' = k^*(z,\mu)\}$ is an indicator function equal to one for $k' = k^*(z,\mu)$ and zero otherwise. Thus, the mass $\int_{\mathcal{K}} G\left(\bar{\kappa}\left(k;z,\mu\right)\right)\mu(dk)$ of the current capital distribution is shifted to $k' = k^*(z,\mu)$. For any other value of k', only the first line in (31) is relevant.

The investment decision characterized in (30) allows to rewrite equilibrium consumption and labor as follows:

$$
\begin{aligned}
C(z,\mu) = \int_{\mathcal{K}} \Bigg[& zF\left(ku_N(k;z,\mu), n_N^f(k;z,\mu)\right)\left(1 - G\left(\bar{\kappa}\left(k;z,\mu\right)\right)\right) \\
& + zF\left(ku_I(k;z,\mu), n_I^f(k;z,\mu)\right) G\left(\bar{\kappa}\left(k;z,\mu\right)\right) \\
& - \left(\gamma k^*(z,\mu) - \left(1 - \delta(u_I(k;z,\mu))\right) k\right) G\left(\bar{\kappa}\left(k;z,\mu\right)\right) \Bigg] \mu(dk),
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
N(z,\mu) = \int_{\mathcal{K}} \Bigg[& n_N^f(k;z,\mu)\left(1 - G\left(\bar{\kappa}\left(k;z,\mu\right)\right)\right) + n_I^f(k;z,\mu) G\left(\bar{\kappa}\left(k;z,\mu\right)\right) \\
& + \int_0^{\bar{\kappa}(k;z,\mu)} \kappa G(d\kappa) \Bigg] \mu(dk).
\end{aligned}
\tag{33}
$$

Uniqueness of the goods and labor market equilibrium is likely to hold, but not guaranteed. Appendix C contains some general considerations on uniqueness in lumpy investment models as well as steady state and simulation results on uniqueness for this paper's model specification and calibration.

3.5 Specification and Calibration

The specification of preferences and technology closely follows the literature on lumpy investment. The model is calibrated to match annual data from Germany. Unfortunately, I do not have access to a dataset with quantitative firm-level data on both utilization and investment.[18] Therefore, I cannot re-calibrate the lumpy investment model for the case of variable utilization. Instead, I choose most parameters according to Bachmann and Bayer (2014), who estimate or calculate many parameters directly from German firm-level or national accounts data.

In accordance with the literature on lumpy investment models (including Thomas, 2002, Khan and Thomas, 2003, 2008, Bachmann et al., 2013, and Bachmann and Bayer, 2014), I assume that the firms' production function takes a Cobb-Douglas form with decreasing returns to scale,

$$
zF(ku,n) = z(ku)^{\theta} n^{\nu}, \qquad \text{with } \theta > 0, \nu > 0, \theta + \nu < 1, \tag{34}
$$

[18] The datasets used in previous studies do not include information on utilization while the dataset used in section 2 of this paper contains utilization, but only qualitative information on investment.

and that the representative household's period utility function is additively separable and linear in labor:

$$U(C, 1-N) = \log(C) + A(1-N). \tag{35}$$

This type of utility function results from the standard indivisible labor model in the spirit of Hansen (1985) and Rogerson (1988). In this model, each individual can either work some given positive number of hours or not at all. Hansen (1985) shows that the representative household in such an economy has a utility function as given in (35).

The depreciation function is specified following Ríos-Rull et al. (2012):

$$\delta(u_t) = \delta_0 + \delta_1 \left(u_t^{1+1/\xi} - 1 \right). \tag{36}$$

With $\delta_1 > 0$ and $\xi > 0$, this depreciation function is increasing and convex in u_t, a common way to model the costs of variable capacity utilization (cf. King and Rebelo, 2000). Moreover, this specification includes fixed capacity utilization as a special case if $\xi \to 0$.

Given the functional forms specified in (34), (35) and (36), some of the optimality conditions derived in section 3.4 can be simplified as follows:

$$p = \frac{1}{C}, \tag{37}$$

$$w = \frac{A}{p}, \tag{38}$$

$$n_I^f = \left(\frac{\nu^{1+\xi(1-\theta)} z^{1+\xi} \theta^{\xi\theta} k^{\theta}}{w^{1+\xi(1-\theta)} \left[\delta_1(1+1/\xi)\right]^{\xi\theta}} \right)^{\frac{1}{1+\xi(1-\theta)-\nu(1+\xi)}}, \tag{39}$$

$$u_I = \left[\frac{\theta z k^{\theta-1} (n_I^f)^{\nu}}{\delta_1(1+1/\xi)} \right]^{\frac{\xi}{1+\xi(1-\theta)}}, \tag{40}$$

$$n_N^f = \left[\frac{\nu z (k u_I)^{\theta}}{w} \right]^{\frac{1}{1-\nu}}, \tag{41}$$

$$u_N = \left[\frac{\gamma \theta p z^{\frac{1}{1-\nu}} k^{\frac{\theta}{1-\nu}-1} \nu^{\frac{\nu}{1-\nu}}}{\delta_1(1+1/\xi)\beta \mathbb{E}\left[V_1^0 \left(\frac{\left(1-\delta_0-\delta_1\left(u_N^{1+1/\xi}-1\right)\right)k}{\gamma}; z', \mu' \right) \right] w^{\frac{\nu}{1-\nu}}} \right]^{\frac{\xi}{1+\xi\left(1-\frac{\theta}{1-\nu}\right)}}. \tag{42}$$

(37) shows that firms value current output using the marginal utility of consumption. The

real wage (38) is determined by the marginal rate of substitution of leisure for consumption. (39) and (40) determine optimal choices of labor and utilization of firms that adjust their capital stock. Labor demand of non-investing firms is characterized by (41). (42) implicitly determines the utilization rate of these firms.

Following the literature (e.g., Thomas, 2002, Khan and Thomas, 2003, 2008, Bachmann et al., 2013, and Bachmann and Bayer, 2014), I assume the fixed costs of investment to be drawn from a uniform distribution $G(\kappa) = \kappa/B$. This assumption is not innocuous.[19] Nevertheless, I rely on it because this facilitates comparing my findings to those of other studies.

My calibration of the parameters heavily relies on Bachmann and Bayer (2014). In particular, the values of the parameters A, B, β, γ, δ_0, θ and ν are identical to their model.[20] Since Bachmann and Bayer (2014) do not consider variable utilization, I rely on other sources for the calibration of $\bar{u}$ and the depreciation function's parameters δ_1 and ξ. As noted by King and Rebelo (2000) and Ríos-Rull et al. (2012), little is known about ξ. I use $\xi = 1$, which approximately corresponds to the point estimate from Basu and Kimball (1997). Following Ríos-Rull et al. (2012), δ_1 is chosen such that the steady state utilization rate equals one in the model without capital adjustment costs. This normalization simplifies the comparison of this paper's model with the frictionless model. Given a steady state utilization of one, $\bar{u}$ is determined using the data from the KOF Swiss Economic Institute analyzed in section 2. Mean firm-level utilization equals 82.3% in the dataset. Maximum feasible utilization is computed as $\bar{u} = 100/82.3 = 1.215$, 21.5% above the frictionless model's steady state. Finally, the Markov chain for aggregate productivity is chosen as an approximation to a continuous AR(1) process with Gaussian white noise innovations:

$$\ln(z') = \rho_z \ln(z) + \varepsilon', \qquad \text{with } \varepsilon' \sim \mathcal{N}(0, \sigma_z^2), |\rho_z| < 1. \tag{43}$$

[19] Indeed, considering a different distribution, Gourio and Kashyap (2007) find macroeconomic effects resulting from fixed capital adjustment costs and investment lumpiness at the firm level. Their preferred calibration features a "compressed" distribution, i.e., many firms bunch around two levels of fixed cost. With many firms facing a similarly sized fixed costs, it is possible that an aggregate shock pushes a lot of firms across the threshold from not investing to investing.

[20] B, the upper bound of the fixed cost distribution, has been calibrated very differently in the literature. E.g., Thomas (2002) and Khan and Thomas (2003, 2008) choose a substantially lower value for B. Nevertheless, the choice of $B = 0.2$ does not appear to be too large because, in the steady state of this paper's model, it leads to expenditure on adjustment costs that amount to approximately 4% of investment spending. This is still considerably lower than suggested in Gourio and Kashyap (2007), who report average adjustment costs of roughly 7.5% of investment based on the findings of Cooper and Haltiwanger (2006).

I rely on the discretization procedure in Tauchen (1986) with nine grid points. The parameters ρ_z and σ_z are calibrated such that, in a simulation of the model, the first-order autocorrelation and the volatility of aggregate output correspond to those of detrended annual GDP of Germany. Table 4 summarizes the parameter choices.

Table 4: Calibration of model parameters

Parameter		Value	Chosen to match/Source
Discount factor	β	0.97	German average real interest rate (Bachmann and Bayer, 2014)
Disutility of labor	A	2	$N = 0.33$ in a model without capital adjustment costs. This corresponds to working one third of the available time.
Output elasticity of labor	ν	0.5565	Share of labor expenditure in value added (Bachmann and Bayer, 2014)
Output elasticity of capital services	θ	0.2075	Share of capital expenditure in value added (Bachmann and Bayer, 2014)
Depreciation rate	δ_0	0.094	Depreciation rate, German national accounting data (Bachmann and Bayer, 2014)
Utilization-dependent depreciation rate	δ_1	0.0697	Normalization of $u = 1$ in the steady state of the model without capital adjustment costs.
Inverse elasticity of $\delta_u(u)$ with respect to utilization	ξ	1	Basu and Kimball (1997)
Maximum feasible utilization rate	$\bar{u}$	1.215	Maximum feasible utilization relative to mean firm-level utilization in the data from the KOF Swiss Economic Institute
Autocorrelation of productivity	ρ_z	0.37	First-order autocorrelation of German GDP (annual, detrended)
Standard deviation of innovations in productivity	σ_z	0.015	Volatility of German GDP (annual, detrended)
Gross economic growth	γ	1.014	Aggregate investment rate, German national accounting data (Bachmann and Bayer, 2014)
Adjustment cost parameter	B	0.2	Skewness and kurtosis of German firms' investment rates (Bachmann and Bayer, 2014)

4 Model Solution

Solving for the competitive equilibrium described in the previous sections is nontrivial. The aggregate state vector includes μ, the distribution of capital across firms. This distribution is nonstandard: It has point masses because, in each period, the mass of investing firms jumps to the same point of the distribution. To address this issue, the common procedure in the lumpy investment literature consists in assuming that agents base their decisions not on the entire distribution, but only on a set of statistics or moments of the distribution.

Previous studies have mostly adopted the method of Krusell and Smith (1997, 1998) for the numerical model solution.[21] However, alternative methods have been proposed to solve incomplete market models with heterogeneous agents and aggregate risk. Den Haan (2010) provides a comparison. He finds that, overall, the algorithm of Reiter (2010) performs best in terms of accuracy. In particular, this algorithm "*clearly performs the best in terms of the accuracy of the individual policy rules and the accuracy of its aggregate law of motion is close to the most accurate aggregate laws of motion*". Although it is not the fastest algorithm, the method of Reiter (2010) still outperforms the Krusell-Smith algorithm in terms of speed. For these reasons, I use the method of Reiter (2010) to solve the model described in this paper.

This algorithm solves the model by backward iteration on a finite grid of points in the aggregate state space. Consistency between the solution of individual firms and the aggregate solution is enforced in each backward iteration step. In contrast to the Krusell-Smith algorithm, the solution method of Reiter (2010) does neither rely on a parameterization of the aggregate law of motion nor on simulations of the model. The latter might be a reason for both the better speed and the accuracy of the algorithm, as problems of sampling errors due to model simulations are avoided.

In the remainder of this section, I provide an overview of the method of Reiter (2010) and its application to the lumpy investment model with variable utilization. For more details, I refer to Reiter (2010) and the literature cited therein.

First, a grid for firm-level capital is specified. I assume that capital lies between zero and 1.15.[22] 100 points within this range are selected, which delivers 99 intervals over which the discrete cross-sectional distribution of capital is defined. Because there is more curvature in the region of low capital, I select smaller intervals in this part of the distribution.

Second, I compute the steady state of the model without aggregate shocks. A steady

[21] Examples include Khan and Thomas (2003, 2008), Bachmann et al. (2013) and Bachmann and Bayer (2014). They approximate the distribution μ by a finite set of its moments and its evolution Γ by a forecasting rule, usually a log-linear rule, that predicts future moments based on current moments of μ and on aggregate productivity. Moreover, a functional form for the equilibrium price is assumed. The numerical solution proceeds in two steps, which are repeated until convergence is achieved. First, conditional on the assumed pricing rule and the conjectured law of motion for the moments of the capital distribution, the dynamic programming problem becomes computable and the firms' value and policy functions can be solved for by value function iteration. Second, given value and policy functions, the economy is simulated without imposing the presumed equilibrium pricing rule. This simulation generates time series for p and moments of μ, which are then used to update the assumed forecasting and pricing rules. Subsequently, the procedure returns to the first step and continues until the forecasting and pricing rules converge.

[22] Zero is a natural lower bound. Larger upper bounds than 1.15 were used, but the steady state fraction of firms with higher capital turned out to be zero. This upper bound is more than 70% larger than the steady state capital stock of 0.66. Note that, in the presence of aggregate shocks, investing firms may choose larger capital stocks than this upper bound. The value function at larger capital levels is computed using extrapolation.

state is reached if the fraction of firms lying in a specific interval of the capital distribution is constant over time, i.e., if the histogram of firm-level capital does not change over time. Solving for the steady state involves the following steps:

(1) Guess the steady state consumption C. Given C, p and w are determined by equations (37) and (38).

(2) Solve for the optimal firm decisions by value function iteration.[23]

(3) Compute the matrix of transition probabilities between the intervals of the capital distribution. If $p(D')$ and $p(D)$ denote next and current period's probability distribution over firm-level capital, then the transition matrix T is characterized by

$$p(D') = Tp(D).$$

(4) Find the steady state distribution D^* as the solution to $p(D^*) = Tp(D^*)$.

(5) Check whether the steady state distribution implies a consumption level consistent with the initial guess C. If not, restart with a different initial guess.

Third, one needs to specify a set of statistics m of the distribution μ which replace μ as state variable. Thus, similar to the method of Krusell and Smith (1997, 1998), the firms are assumed to base their decisions only on a few statistics m rather than the entire distribution μ. For computational feasibility, this paper uses only the first moment of the capital distribution.[24]

Forth, a reference distribution $D_R(z, m)$ is specified. This is a guess of what the distribution should approximately look like if the aggregate state is (z, m). Following Reiter (2010), I use a scaled version of the steady state distribution without shocks as reference distribution. The scale factor is chosen such that the reference distribution exactly satisfies the first moment condition, i.e., $\mathbb{E}_{D_R(z,m)}[k] = m_1$.

Fifth, a proxy distribution $D_P(z, m)$ is chosen. This step selects the distribution that is closest (in a mean square sense) to the reference distribution and exactly satisfies the moment conditions. Of course, this step was only necessary if m would include more than the first

[23]The value and policy functions are solved for at 99 grid points corresponding to the midpoints $\mathscr{K}_j$ of the intervals specified above.

[24]This practice is quite common in the literature. Bachmann et al. (2013) and Bachmann and Bayer (2014), for example, also rely on the aggregate capital stock only.

moment of the capital distribution because the reference distribution already exactly satisfies $\mathbb{E}_{D_R(z,m)}[k] = m_1$.

Sixth, the model is solved by backward iteration. This involves the following steps:

(1) Initialize next period's value function $V^0(k'; z', m')$ by the steady state value function for all z' and m'.

(2) For any point (z, m) in the grid of aggregate states and for any value of z', find the equilibrium values of m'. This requires the following substeps:

(2.1) Guess m' and p. Given p, w is determined by equation (38).

(2.2) Use an interpolation scheme to obtain the value function off the grid for m'. I use the shape-preserving quadratic spline of Schumaker (1983) (see also Judd, 1998). This method also provides an algorithm to obtain estimates of the slope of the value function.

(2.3) Compute optimal labor, utilization and capital choices of investing firms starting from the proxy distribution $D_P(z, m)$. Equations (39) and (40) yield the closed-form solution for n_I^f and u_I as a function of k, z and w. Given V' and the guess for p, (24) determines the optimal capital choice k'.[25]

(2.4) Compute optimal labor and utilization choices of non-investing firms starting from the proxy distribution $D_P(z, m)$. Computations are more involved than for investing firms. An essential element that speeds up the solution is the use of the endogenous grid point method of Carroll (2006). The basic idea is to formulate a grid for k' rather than k. Let $\mathscr{K}_j$, $j = 1, \ldots, n_k$, denote the grid points for firm-level capital. Using the firms' optimality conditions, one can deduce the capital levels $\tilde{k}_j$ at which it is optimal to choose a capacity utilization leading to $k'_j = \mathscr{K}_j$. The value function at the endogenous grid points $\tilde{k}_j$ can then be computed without actually solving a maximization problem because the grid points are chosen such that $k'_j = \mathscr{K}_j$ is the optimal outcome. Finally, Schumaker splines are used to obtain the value of non-investing firms at the grid points of the proxy distribution instead of $\tilde{k}_j$.

[25]The next period's value function V' is known from the previous backward iteration step or, in the first iteration, from the steady state.

(2.5) Substeps (2.3) and (2.4) yield the value of investment and non-investment at the grid points of the proxy distribution. Equation (30) then determines the threshold for the binary investment decision at each of these grid points.

(2.6) Compute aggregate variables. Check whether the resulting p and m' are consistent with the guess from substep (2.1). If not, restart with a different initial guess.

(2.7) From the previous substeps, the value $\tilde{V}(k; z, m, z', m'(z, m, z'))$ is obtained at each grid point for k of the proxy distribution. Schumaker splines are then used to compute the value at the original grid points $\mathscr{K}_j$.

(3) Update the value function using $\tilde{V}(k; z, m, z', m'(z, m, z'))$ from the previous step. For each $\mathscr{K}_h$, z and m, the updated value function is given by

$$V(\mathscr{K}_h; z = z_i, m) = \sum_j \pi_{ji} \tilde{V}(\mathscr{K}_h; z = z_i, m, z' = z_j, m'(z_i, m, z' = z_j)). \tag{44}$$

Steps (2) and (3) of this backward iteration are repeated until convergence in the value function is achieved. I use the criterion that the absolute difference between the value functions of two consecutive iterations is at most 10^{-6} for all grid points.

As pointed out by Reiter (2010), computation can be accelerated by not solving for $m'(z, m, z')$ in every iteration. After some full iterations consisting of steps (2) and (3), $m'(z, m, z')$ is only computed every few iterations with some intermediate "acceleration iterations" which use $m'(z, m, z')$ of the previous full iteration.

5 Results

This section presents the results from the lumpy investment model with variable capacity utilization, starting with a description of the steady state, followed by a characterization of optimal firm-level decisions on investment, utilization and labor, and closing with a description of the macroeconomic implications thereof. The results of this paper's model are compared with those of three other models which differ in either or both of the following two dimensions: whether utilization is variable and whether capital adjustment entails fixed costs. Thus, four models are compared: this paper's *variable utilization lumpy investment model* (VULIM), the *standard lumpy investment model* (SLIM) with fixed utilization, a *variable utilization frictionless model* (VUFM) without fixed capital adjustment costs, and the *standard frictionless* *model* (SFM) with neither variable utilization nor fixed costs.

5.1 Steady state

Figure 5 shows the distribution of firm-level capital in the steady state without aggregate shocks. The fraction of firms owning a certain capital stock is increasing in capital. The figure also plots the *adjustment hazard*, i.e., the fraction of firms paying the fixed costs and adjusting capital to the target level. Clearly, the model features the increasing hazard property, i.e., the probability of investment increases with the distance from the target capital stock.[26] The reason for the increasing hazard becomes apparent in figure 6, which depicts the value of investing (excluding the fixed costs) and the value of inaction. The farther below the target level a firm's capital stock is, the more the value of investment exceeds the value of inaction. Thus, the willingness to adjust the capital and incur the fixed costs increases.

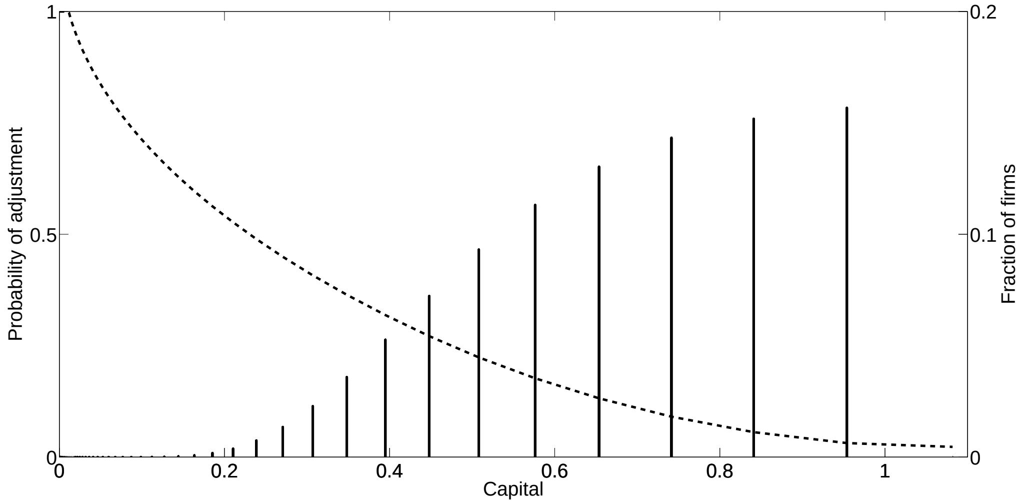


Figure 5: Steady state distribution of firm-level capital (bars, right axis) and probability of capital adjustment (dashed line, left axis).

Table 5 provides a comparison of the different models' steady state. The ratios of aggregate labor and consumption to output are almost identical across all models. The capital to output ratio, however, is larger in the models without capital adjustment costs (VUFM and SFM). In the VULIM, the aggregate steady state utilization rate is considerably higher than in the other models while the capital to output ratio is smaller. Table 5 also reveals that, compared

[26]This feature has been well documented in the literature (examples include Caballero et al., 1995, Caballero and Engel, 1999, Cooper et al., 1999, Thomas, 2002, Khan and Thomas, 2003, 2008, and Bachmann et al., 2013).

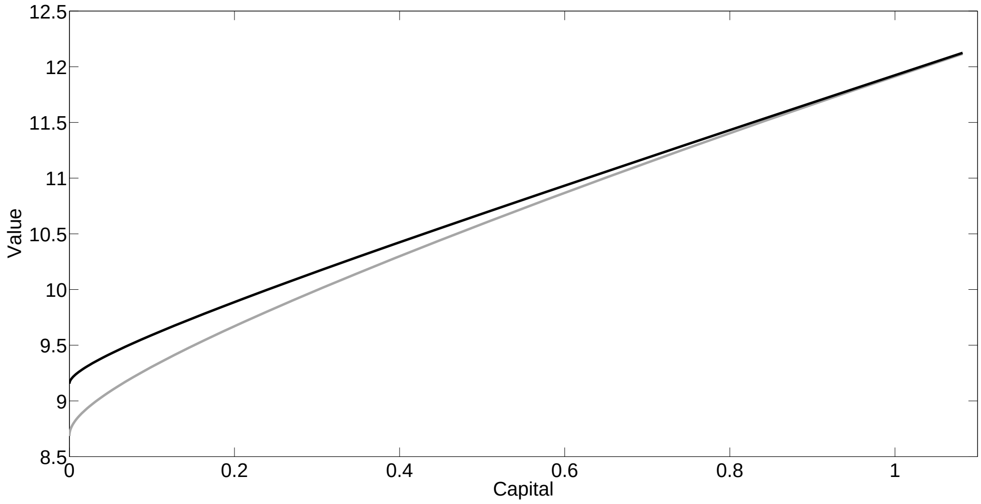


Figure 6: The value of investment excluding fixed costs (black line) and the value of not investing (gray line).

to the SLIM, fewer firms invest in the VULIM. Moreover, there are some differences in the firm-level investment rate (i/k) distribution. In the VULIM, the mean and standard deviation slightly exceed the respective moments for the SLIM. The most marked difference pertains to the mean investment rate of investing firms, which is substantially larger in the VULIM.

Table 5: Steady state

	VULIM	SLIM	VUFM and SFM
K/Y	1.32	1.36	1.49
N/Y	0.66	0.67	0.65
C/Y	0.84	0.84	0.84
U	1.09	1	1
Fraction of investing firms	0.16	0.18	1
Mean of i/k	0.20	0.19	0.11
Standard deviation of i/k	0.57	0.54	-
Mean of i/k if $i/k > 0$	1.26	1.06	0.11

Notes: Capital letters denote aggregate variables. N only includes labor used for production, not the fixed costs denominated in hours of labor. The models VUFM and SFM yield the same outcomes because δ_1 is calibrated such that steady state utilization is equal to one in the model without adjustment costs.

5.2 Firm-level investment, utilization and labor decisions

5.2.1 Adjustment hazard

Figure 7 shows how aggregate productivity affects the probability of investment. The adjustment hazard is computed at the smallest, middle, and largest of the nine productivity states.[27] The probability of investment is clearly increasing in productivity. The difference in the adjustment hazard curves between the lowest and the highest productivity state amounts up to 17 percentage points. Moreover, the target capital level is increasing in productivity. At low aggregate productivity states, high capital firms with tiny fixed costs optimally undertake negative investment because their capital stock sufficiently exceeds the desired level. In contrast, no firm wants to actively decrease its capital at the medium and higher productivity states.

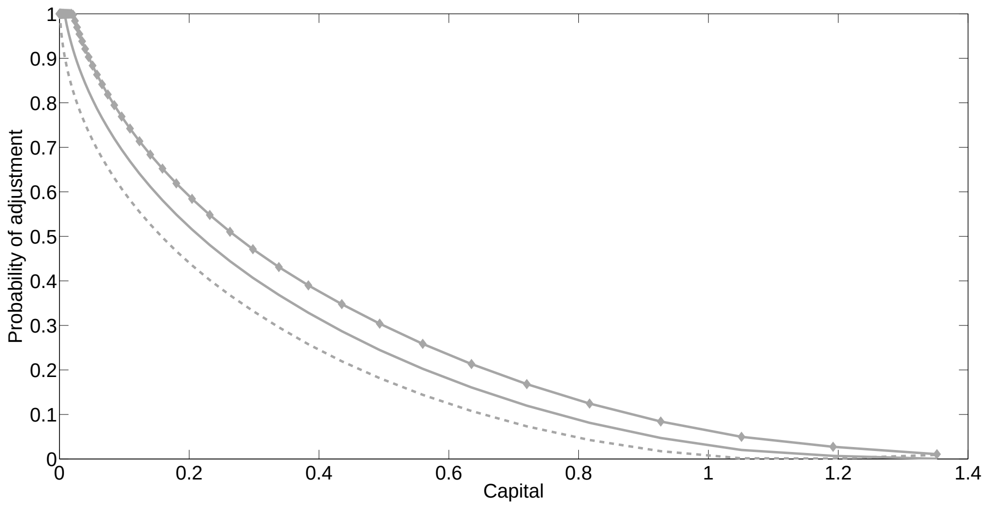


Figure 7: The adjustment hazard of the VULIM at different levels of aggregate productivity: the lowest productivity state (dashed line), the medium state (solid line), and the highest state (solid line with markers).

To analyze the importance of variable utilization for firms' investment decisions, figure 8 compares the adjustment hazard of the SLIM and the VULIM for different productivity states. The adjustment hazard of the SLIM fluctuates to a smaller extent across the cycle. Consequently, there is a cyclical difference in the probability of investment between the two

[27]To isolate the impact of productivity, I calculate the probability of investment at the identical firm-level capital distribution for different productivity states. Specifically, I calculate it at the proxy distribution $D_P(z, \bar{m})$ (cf. section 4) for average moments $\bar{m}$, where $\bar{m}$ is computed as a simulation average (note that $\bar{m}$ differs between the four models VULIM, SLIM, VUFM and SFM). For each model, I simulate the economy over 5200 periods, starting from the steady state without aggregate shocks. The first 200 periods are discarded to reduce the impact of the initial productivity state and capital distribution.

models.[28] At low states of aggregate productivity, the VULIM features a smaller probability of investment for almost any firm-level capital stock. At high productivity states, firms with large capital stocks are more likely to invest in the VULIM while low capital firms still have a smaller probability of adjusting capital.

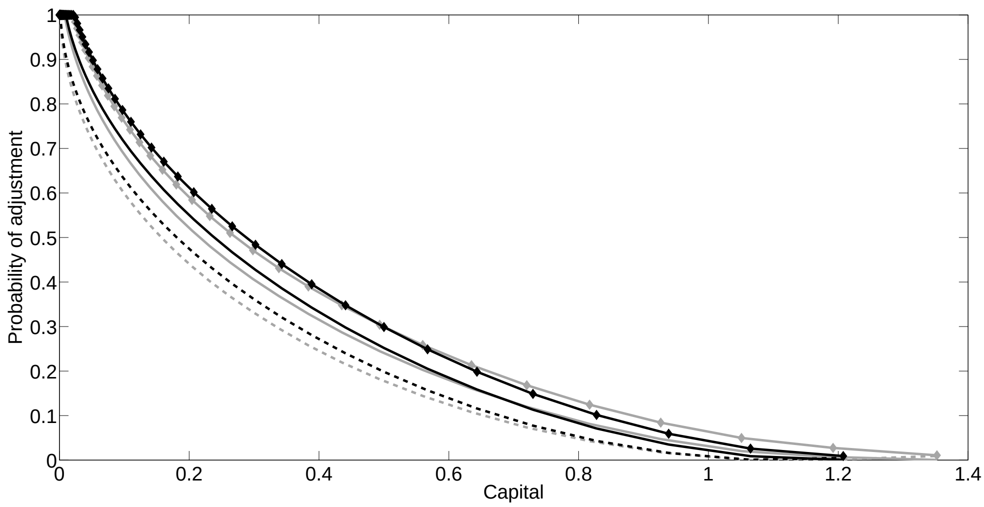


Figure 8: The adjustment hazard of the VULIM (gray lines) and the SLIM (black lines) at different levels of aggregate productivity: the lowest productivity state (dashed lines), the medium state (solid lines), and the highest state (solid lines with markers).

5.2.2 Capacity utilization

The utilization decision is conceptually very different for investing and non-investing firms. For the former, it is an intratemporal choice that is comparable to the decision on labor demand. For the latter, however, it is an intertemporal decision. Figure 9 plots the utilization rate of investing and non-investing firms for average aggregate productivity and mean moments of the capital distribution. Except for very low or high capital levels, non-investing firms choose a substantially lower utilization rate.[29] This lower rate allows the firms to save capital for future years, i.e., to smooth capital over the periods until the firm adjusts its capital stock for the next time.

The utilization function of both investing and non-investing firms is decreasing in cap-

[28]This finding is in line with the empirical evidence presented in section 2 which reveals a cyclical difference in the predicted fraction of investing firms when utilization is variable or forced to be constant.

[29]This finding is consistent with the data analyzed in section 2. In a fixed effects regression of utilization on the investment dummy, real GDP growth and an interaction term, the coefficient of the investment dummy is significantly positive. The results indicate that investing firms ceteris paribus have a 4.6 percentage points higher utilization rate than non-investing firms.

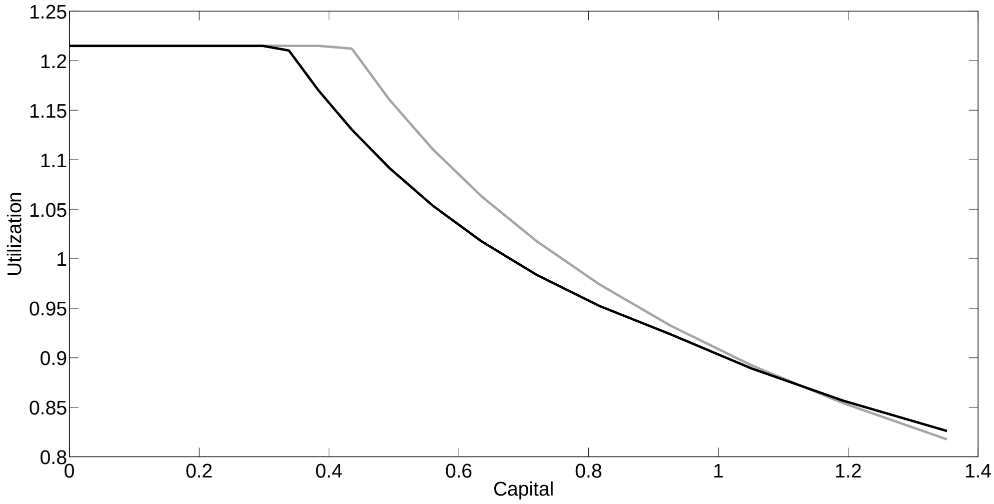


Figure 9: Utilization of investing (gray line) and non-investing (black line) firms.

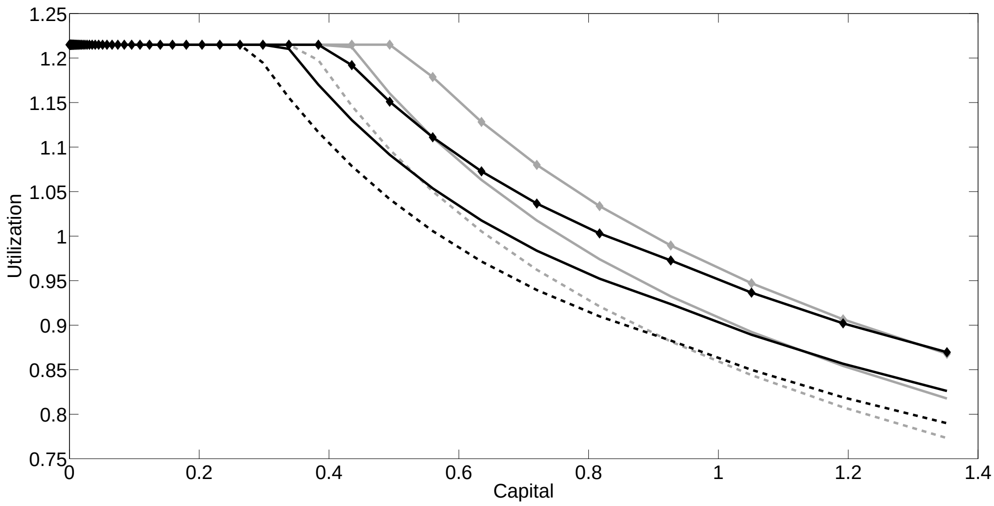


Figure 10: Utilization of investing (gray lines) and non-investing (black lines) firms at different levels of aggregate productivity: the lowest productivity state (dashed lines), the medium state (solid lines), and the highest state (solid lines with markers).

ital. Note that firms with a lot of capital, i.e., firms which have recently invested, choose a utilization rate that lies substantially below the maximum feasible rate. Thus, investing firms adjust their capital stock to an extent such that they have reserve capacity.[30] There is an intuitive explanation for that: Forward-looking firms facing fixed capital adjustment costs jointly plan their investment and utilization paths. In the light of trend productivity growth, it is optimal to invest up to a capital stock that is too large (i.e., not fully utilized) in the short run, but allows to keep up with economic growth for some more years. Moreover, reserve capacity building is optimal because of depreciation: If there were not any reserve capacity, depreciation would soon cause the newly adjusted capital stock to fall below the desired level. These considerations highlight the inherent interrelation between firm-level investment and capacity utilization decisions, which is one of the main motivations for this paper.

Figure 10 depicts the utilization rate of investing and non-investing firms for different aggregate productivity states. Utilization of both types of firms is increasing in productivity. For high capital stocks, the utilization rate of non-investing firms can exceed the one of investing firms. This is the case for firms whose capital stock is close to or exceeds their target level. Intuitively, because utilization is an intertemporal decision for non-investing firms, these firms have an incentive to utilize their capital in a way to approach or remain close to their target level. Compared to investing firms, this incentive entails a larger utilization rate for firms whose current capital is close to or exceeds the target level and a lower utilization otherwise.

Finally, a comparison reveals that capacity utilization of investing firms is very similar in the lumpy investment model and the frictionless model (see figure 19 in the appendix).

5.2.3 Labor demand

The labor demand of both investing and non-investing firms is determined by the optimality condition $w = zF_2(ku, n)$. Consequently, their labor demand (for a given k) only differs if they choose different utilization rates. Since investing firms have a larger utilization rate in many cases (as discussed in section 5.2.2), their labor demand usually also exceeds the demand of non-investing firms.

Figure 11 compares the labor demand of the VULIM and the SLIM for average aggregate productivity and mean moments of the capital distribution. The labor demand of firms in the

[30] The results in section 5.2.4 show that the target capital of investing firms is indeed larger in the VULIM than in the SLIM.

VULIM is larger for small levels of firm capital while it is smaller for high capital levels. This finding corresponds to the discussion in section 5.2.2: In the model with variable utilization, adjusting firms invest up to a large capital stock which is not immediately fully utilized. Therefore, labor demand is smaller for large levels of capital. As the capital diverges from the target due to depreciation and technological progress, both utilization and labor demand increase relatively to the SLIM.

A comparison of the lumpy investment models (VULIM and SLIM) with the corresponding frictionless models (VUFM and SFM) reveals that labor demand is very similar for the investing firms (see figures 20 and 21 in the appendix).

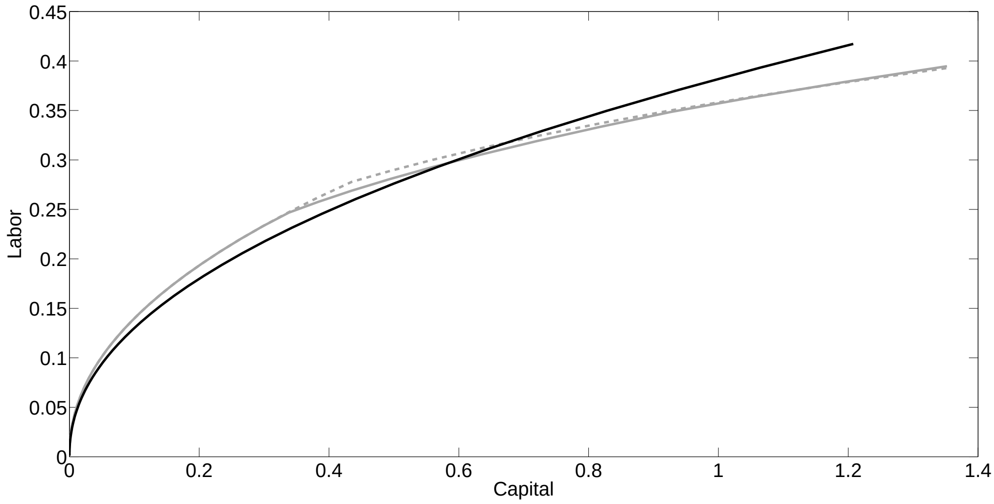


Figure 11: Labor demand of the VULIM (gray lines) and the SLIM (black line). The dashed line indicates investing firms, solid lines depict non-investing firms. In the SLIM, the labor demand of both types of firms coincides.

5.2.4 Investment decisions and investment rate distribution

In the model with variable utilization, investing firms build up reserve capacity, i.e., they adjust capital such that the new stock is not fully utilized. As a consequence, the target capital level is larger than in the model with constant utilization. This finding holds for all states of productivity, but particularly for medium and high states. Thus, the difference between the target capital of the VULIM and the SLIM is pro-cyclical.

Table 6 shows the target capital and the investment of adjusting firms for both models and all states of productivity.[31] In addition, the table presents the fraction of firms adjusting

[31]The quantities in table 6 are computed at the proxy distribution $D_P(z, \bar{m})$ for average moments $\bar{m}$, where

their capital stock. In any productivity state, the VULIM features fewer of those firms. Thus, the SLIM overestimates the average fraction of adjusting firms. In contrast, it underestimates the cyclical variability in this fraction. This finding is in line with the empirical evidence presented in section 2. Overall, table 6 suggests that investment is more lumpy if capacity utilization is variable because there are fewer adjusting firms, but the investing firms adjust up to a higher target capital.

In the frictionless models, the target capital is substantially smaller than in the lumpy models (see table 12 in the appendix) because the firms can readjust their capital stock in every period without incurring fixed costs. There are only small differences between the target capital levels of the VUFM and the SFM. Thus, the fact that variable utilization leads to larger target capital levels of investing firms specifically pertains to the lumpy investment models. This is related to the incentive for rare and large investments in the lumpy models and the reserve capacity building, which only comes into complete effect if utilization is allowed to vary.

Table 6: Investment decisions as a function of aggregate productivity

Productivity	VULIM			SLIM		
	Fraction active	Target capital	Conditional investment	Fraction active	Target capital	Conditional investment
$z = 0.93$	0.070	1.02	0.54	0.094	0.98	0.51
$z = 0.95$	0.078	1.10	0.61	0.104	1.00	0.52
$z = 0.97$	0.087	1.16	0.66	0.112	1.04	0.56
$z = 0.98$	0.097	1.23	0.72	0.119	1.09	0.60
$z = 1.00$	0.107	1.28	0.76	0.127	1.13	0.64
$z = 1.02$	0.117	1.34	0.81	0.136	1.17	0.67
$z = 1.03$	0.127	1.39	0.85	0.144	1.21	0.70
$z = 1.05$	0.138	1.44	0.89	0.153	1.24	0.73
$z = 1.07$	0.148	1.48	0.93	0.161	1.28	0.77

Notes: *Fraction active* denotes the fraction of firms adjusting their capital stock, thereby incurring fixed costs. *Target capital* is the capital level that active firms select. *Conditional investment* denotes the investment conditional on being active, i.e., the investment of adjusting firms.

The distribution of the firm-level investment rates (i/k) is characterized in table 7. The table entries are obtained by simulating the VULIM and the SLIM over 5200 periods starting from the steady state distribution. I use the identical simulation of the aggregate productivity series for both model simulations. Differences in the investment rate distribution are therefore only caused by different model properties (i.e., variable or fixed utilization). The first 200 periods are discarded to reduce the impact of the initial productivity state and capital

$\bar{m}$ is computed as a simulation average as described in section 5.2.1. Note that $\bar{m}$ is model-dependent.

distribution.

The resulting distribution of the investment rates features a higher mean, standard deviation, skewness and kurtosis in the VULIM. Thus, firm-level investments relative to the capital stock are larger on average and more lumpy. Moreover, the rise in the skewness highlights that variable utilization magnifies the asymmetry of investment rates.

The moments of the investment rate distribution show a clear dependence on aggregate productivity. The mean and the standard deviation increase with productivity, the skewness and the kurtosis decrease. The positive correlation between the standard deviation of investment rates and the business cycle fits recent empirical evidence: Bachmann and Bayer (2014) have established the pro-cylicality of the firm-level investment rate dispersion.

In the frictionless models, all firms are identical and have the same investment rate. Table 13 in the appendix shows that this investment rate of the VUFM and SFM amounts to approximately half of the mean investment rate of the lumpy models.

Table 7: Moments of the investment rate distribution as a function of aggregate productivity

Productivity	VULIM				SLIM			
	Mean	Std. dev.	Skewness	Kurtosis	Mean	Std. dev.	Skewness	Kurtosis
$z = 0.93$	0.16	0.63	5.6	47.0	0.16	0.51	4.3	26.0
$z = 0.95$	0.18	0.67	5.4	44.1	0.17	0.54	4.2	24.9
$z = 0.97$	0.18	0.68	5.4	43.4	0.17	0.54	4.1	24.5
$z = 0.98$	0.19	0.69	5.3	42.4	0.18	0.56	4.1	24.1
$z = 1.00$	0.20	0.71	5.2	41.3	0.18	0.57	4.0	23.6
$z = 1.02$	0.20	0.73	5.2	40.4	0.19	0.58	4.0	23.2
$z = 1.03$	0.21	0.75	5.1	38.9	0.19	0.59	3.9	22.6
$z = 1.05$	0.22	0.75	5.1	38.8	0.20	0.60	3.9	22.4
$z = 1.07$	0.22	0.76	5.0	38.3	0.20	0.60	3.9	22.0

5.3 Macroeconomic implications

Section 5.2 has documented various effects of variable utilization on firms' optimal decisions. In comparison to the standard lumpy investment model, firms on average (i) adjust their capital stock less frequently, (ii) have a higher target capital level and therefore invest more once they adjust capital, (iii) feature higher utilization and labor demand if their capital stock is small, and (iv) choose smaller utilization and labor if their capital stock is large. (i) and (ii) are suggestive of amplified investment lumpiness compared to the SLIM. This is also reflected in the moments of the investment rate distribution: The mean, standard deviation, skewness, and particularly the kurtosis in the VULIM exceed the respective moments in the SLIM.

In addition to differences between the VULIM and the SLIM on average, the results on the firm-level decisions reveal some cyclical differences. First, the models' adjustment hazards differ more markedly in low productivity states. Second, the SLIM underestimates the cyclical variability of both the target capital level and the fraction of adjusting firms. Finally and obviously, utilization depends on the state of productivity in the VULIM while it is restricted to be constant in the SLIM.

In general, the lumpiness at the micro level may be smoothed by aggregation and general equilibrium price movements. This section analyzes to what extent this holds for the SLIM and the VULIM, where the latter features magnified lumpiness arising from variable utilization. Section 5.3.1 compares moments of aggregate, macroeconomic quantities from the four models VULIM, SLIM, VUFM and SFM. Section 5.3.2 presents impulse response functions to aggregate productivity shocks.

5.3.1 Moments of macroeconomic quantities

The following discussion of moments of macroeconomic aggregates focuses on the differences between the four models rather than on the magnitude of the moments themselves. The model introduced in this paper is too simple to aim for producing realistic aggregate moments.[32] For this reason and because of data unavailability[33], this paper's model is not calibrated to fit certain business cycle properties. Instead, as described in section 3.5, many parameter values are adopted from Bachmann and Bayer (2014), who estimate or calculate most parameters from national accounts or firm-level data.

This section analyzes whether the magnified lumpiness in the VULIM, which is caused by the building up of reserve capacity when utilization is variable, has macroeconomic consequences. Table 8 presents the mean of macroeconomic quantities obtained from a simulation of the four models.[34] The lumpiness of the SLIM and the magnified lumpiness of the VULIM is apparent in the higher target capital and the larger investment of those firms which actually adjust their capital. However, this lumpiness has only negligible effects on aggregate output,

[32] For instance, this model considers only one aggregate shock and firm heterogeneity along the capital dimension. Other models have considered investment-specific shocks (Khan and Thomas, 2003), firm heterogeneity in productivity (Khan and Thomas, 2008; Bachmann et al., 2013; Bachmann and Bayer, 2014), sectoral productivity shocks (Bachmann et al., 2013), or time-varying idiosyncratic productivity risk (Bachmann and Bayer, 2014). The extension of the lumpy investment model by variability in capacity utilization, however, has required a reduction of model complexity in other dimensions for reasons of computational feasibility.

[33] A proper calibration of the VULIM would require a dataset with quantitative firm-level data on both utilization and investment.

[34] The models are simulated over 5200 periods starting from the steady state distribution. The identical simulation of the aggregate productivity series is used for all models. To reduce the impact of the initial capital distribution and productivity state, the first 200 periods are discarded.

consumption, investment, and labor. The VULIM features a larger aggregate capital stock though. Moreover, there is a smaller fraction of firms adjusting their capital. The simulation results for the VULIM also show that, on average, investing firms choose a considerably higher utilization rate than non-investing firms.

Table 8: Mean of macroeconomic aggregates

Variable	VULIM	SLIM	VUFM	SFM
Y	0.51	0.51	0.51	0.51
C	0.43	0.43	0.43	0.43
I	0.08	0.08	0.08	0.08
N	0.33	0.33	0.33	0.33
N investing	0.29	0.29		
N non-investing	0.34	0.34		
K	0.83	0.76	0.76	0.76
Target capital	1.25	1.12	0.76	0.76
I if investing	0.77	0.63	0.08	0.08
I/K	0.10	0.11	0.11	0.11
U	0.98		1.00	
U investing	1.11			
U non-investing	0.96			
Fraction active	0.11	0.13	1.00	1.00

Table 9 presents the volatility of macroeconomic aggregates. Variable utilization appears to have a considerable impact. The variables characterizing investment lumpiness are more volatile in the VULIM than in the SLIM: The standard deviation of the target capital, the investment of adjusting firms and the fraction of those firms is larger both in absolute terms and relative to output. Also, the standard deviation of Y, I, N, K, and I/K rises, whereas consumption becomes less volatile. However, this impact of variable utilization does not specifically pertain to the lumpy model as a similar pattern can be observed comparing the VUFM and the SFM. For example, consider the relative standard deviation of consumption to investment. It is substantially lower in the VULIM than in the SLIM (0.39 instead of 0.56). Thus, the VULIM achieves to relax the tight link between consumption and investment dynamics in the SLIM which has been mentioned as a potential reason for the aggregate irrelevance of microeconomic lumpiness (cf. Bachmann and Ma, 2012). However, a similar decrease in the relative standard deviation of consumption to investment is also obtained if utilization is allowed to vary in the frictionless model.

The contemporaneous correlations between various macroeconomic series and output are shown in table 10. For many variables, the differences between the VULIM and the SLIM are small. Notable exceptions are the labor demand of adjusting firms and the fraction of those

Table 9: Absolute and relative (to output) standard deviations of macroeconomic aggregates

Variable	VULIM		SLIM		VUFM		SFM	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Y	1.71		1.38		1.83		1.40	
C	0.52	0.31	0.54	0.39	0.54	0.30	0.56	0.40
I	1.34	0.79	0.97	0.70	1.52	0.83	1.02	0.73
N	0.83	0.49	0.59	0.43	0.95	0.52	0.63	0.45
N investing	0.99	0.58	0.72	0.52				
N non-investing	0.87	0.51	0.63	0.46				
K	2.86	1.67	2.15	1.56	2.74	1.50	1.96	1.40
Target capital	6.80	3.98	4.98	3.61	2.74	1.50	1.96	1.40
I if investing	5.55	3.25	4.37	3.17				
I/K	1.69	0.99	1.32	0.96	2.08	1.14	1.38	0.99
U	1.75	1.02			2.36	1.29		
U investing	1.16	0.68						
U non-investing	1.67	0.98						
Fraction active	1.09	0.64	0.85	0.62				

Notes: Columns (1), (3), (5) and (7) contain the standard deviation (multiplied by 100 for readability). The other columns contain the relative standard deviation compared to output. I *if investing* denotes the investment conditional on adjusting capital. Labor N and utilization U are listed both as aggregate and separately for investing and non-investing firms.

firms, both of which are more markedly pro-cyclical in the model with variable utilization. In contrast, consumption and, in particular, aggregate capital feature a considerably lower correlation with output when utilization is variable. Compared to the frictionless models, consumption, capital and the target capital level are more strongly correlated with output in the lumpy models.

Finally, table 11 contains the first-order autocorrelation of the macroeconomic quantities. Many series feature a quite similar persistence in the VULIM and the SLIM, except for output, the target capital and the investment of adjusting firms, all of which are less persistent in the VULIM. In the frictionless models, the series tend to be more weakly autocorrelated with the exception of consumption and, in particular, target capital.

5.3.2 Impulse response functions

This sections presents the effects of aggregate technology shocks on macroeconomic quantities for different models. The impulse response functions are obtained from simulations which start at the steady state distribution. Aggregate productivity is set to the medium state ($z = 1$) in $t = 0$. The technology shock consists in a switch to the aggregate productivity state $z = \tilde{z}$ in $t = 1$. The subsequent evolution of aggregate productivity is simulated 300 times. Let $\hat{X}^{(i)}_{\tilde{z},t}$

Table 10: Correlation of macroeconomic aggregates with output

Variable	VULIM	SLIM	VUFM	SFM
C	0.78	0.84	0.67	0.79
I	0.97	0.95	0.96	0.94
N	0.95	0.92	0.95	0.90
N investing	0.86	0.72		
N non-investing	0.95	0.93		
K	0.18	0.30	0.13	0.28
Target capital	0.96	0.97	0.59	0.73
I if investing	0.98	0.98		
I/K	0.90	0.86	0.90	0.85
U	0.69		0.65	
U investing	0.48			
U non-investing	0.67			
Fraction active	0.86	0.69		

Table 11: First-order autocorrelation of macroeconomic aggregates

Variable	VULIM	SLIM	VUFM	SFM
Y	0.35	0.42	0.31	0.39
C	0.77	0.79	0.84	0.82
I	0.27	0.28	0.22	0.22
N	0.27	0.26	0.22	0.19
N investing	0.26	0.23		
N non-investing	0.28	0.28		
K	0.91	0.90	0.88	0.86
Target capital	0.42	0.50	0.88	0.86
I if investing	0.41	0.50		
I/K	0.26	0.24	0.22	0.19
U	0.42		0.36	
U investing	0.71			
U non-investing	0.42			
Fraction active	0.32	0.34		

denote the time series in period t which results from simulation number i for an initial shock to state $\tilde{z}$. The impulse response at period t is given by

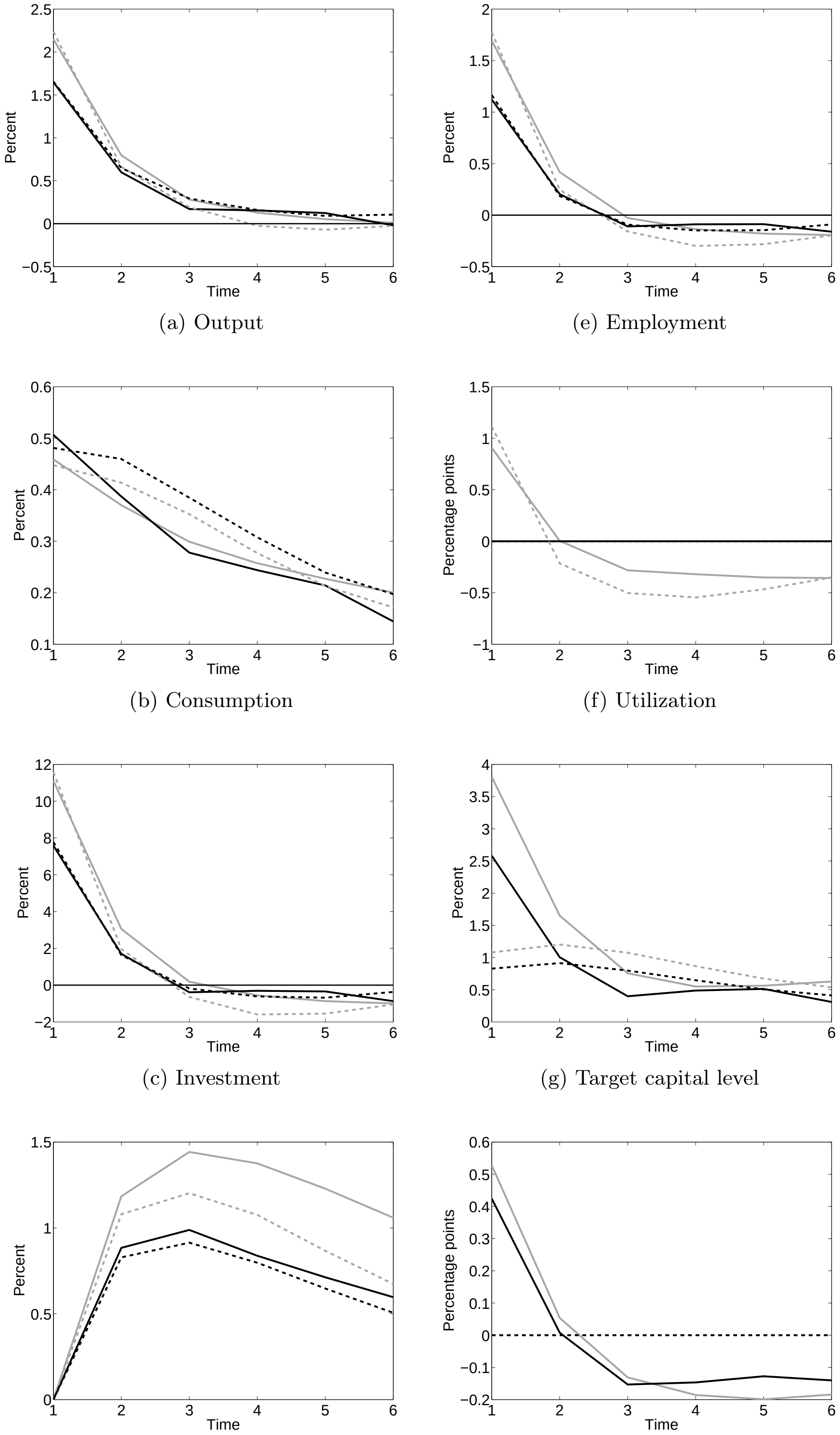
$$IRF_{\tilde{z},t} = \frac{1}{300} \sum_{i=1}^{300} \left(\hat{X}^{(i)}_{\tilde{z},t} - \hat{X}^{(i)}_{1,t} \right) \quad \text{for } \tilde{z} \in \{0.93, 0.95, 0.97, 0.98, 1.02, 1.03, 1.05, 1.07\}. \tag{45}$$

Thus, the impulse response corresponds to the average difference in the simulated macroeconomic time series if aggregate productivity switched to state $z = \tilde{z}$ instead of staying at state $z = 1$ in period $t = 1$.

Figure 12 shows the responses of output, consumption, investment, capital, employment, utilization, target capital, and the fraction of adjusting firms to an aggregate productivity shock of one percent.[35] The responses are depicted in percentage deviations from steady state except for utilization and the fraction of adjusting firms, whose responses are plotted in percentage points. The magnified lumpiness of the VULIM is apparent in panels (g) and (h) of figure 12. The initial increase in the target capital level is almost 50% higher than in the SLIM. Moreover, the fraction of adjusting firms increases more strongly. As a result, aggregate investment and capital respond more markedly to a positive productivity shock. The same holds for employment and output. In contrast, the impulse response of consumption is rather similar across the VULIM and the SLIM.

Thus, apart from consumption, variable utilization leads to substantial differences in the responses of macroeconomic time series to technology shocks. However, this does not necessarily imply that investment lumpiness at the micro level has substantial macroeconomic consequences because the impulse responses depend on utilization variability in a frictionless world as well. Indeed, a comparison of the VULIM with the corresponding frictionless model (VUFM) reveals that the differences in some impulse responses are relatively minor. In particular, the initial responses of output, consumption, investment and employment hardly differ across these models. However, pronounced differences between the VULIM and the VUFM pertain to capital, utilization and the target level. The response of aggregate capital is larger

[35] More precisely, the impulse response functions are estimated for a switch from state $z = 1$ to $z = 1.03$, but scaled down to a productivity shock of one percent for reasons of readability. In principle, there may be non-linearities, i.e., the response may depend non-proportionally on the size of the initial productivity change. However, the responses to different magnitudes of the initial shock turn out to be roughly similar. In addition, the impulse responses may be different depending on the shock being positive or negative. However, the differences turn out to be small for most of the series.



(a) Output
(e) Employment
(b) Consumption
(f) Utilization
(c) Investment
(g) Target capital level
(d) Capital
(h) Fraction of adjusting firms

Figure 12: Impulse responses to a one percent positive technology shock occurring at $t = 1$ for various models: VULIM (gray solid lines), SLIM (black solid lines), VUFM (gray dashed lines), and SFM (black dashed lines).

and more persistent in the VULIM while aggregate utilization initially increases to a smaller extent. The initial increase in the target capital level is more than three times larger. Thus, while positive productivity shocks enhance investment lumpiness at the micro level as investing firms adjust up to a higher target, the overall macroeconomic effects of lumpy investment are small: Few firms invest approximately the same amount that would have been invested by all firms otherwise.

The four models compared in figure 12 differ in the dimensions of utilization variability and capital adjustment costs. In terms of the size of the responses, the variability of utilization appears to be the more relevant dimension. Except for the target capital and the fraction of adjusting firms, the initial magnitude of the impulse response differs more between the models with variable and fixed utilization than between the models with and without fixed capital adjustment costs.

The previous discussion on the impulse responses has been confined to economies starting at the steady state distribution. The remainder of this section addresses the variation of these responses over the business cycle. Following Caballero and Engel (1993) and Bachmann et al. (2013), a *responsiveness index* (RI) is computed. This index measures the response upon impact to an aggregate technology shock as a function of the economy's aggregate state (z_t, μ_t). It is defined as follows:

$$RI_{t,X} \equiv \frac{1}{2}\left[\mathcal{I}_X^+(z_t,\mu_t) - \mathcal{I}_X^-(z_t,\mu_t)\right], \tag{46}$$

where X denotes the series for which the response is computed and where

$$\mathcal{I}_X^+(z_t,\mu_t) \equiv \hat{X}(z_t^+,\mu_t) - X(z_t,\mu_t), \tag{47}$$

$$\mathcal{I}_X^-(z_t,\mu_t) \equiv \hat{X}(z_t^-,\mu_t) - X(z_t,\mu_t). \tag{48}$$

$X(z_t,\mu_t)$ denotes the value of X in period t obtained from the simulation over 5200 periods described above. $\hat{X}(z_t^+,\mu_t)$ and $\hat{X}(z_t^-,\mu_t)$ denote the values of X if aggregate productivity had amounted to z_t^+ and z_t^- instead of z_t, respectively, where z_t^+ and z_t^- denote the adjacent productivity states of z_t.[36] Potential asymmetries between $\mathcal{I}_X^+(z_t,\mu_t)$ and $\mathcal{I}_X^-(z_t,\mu_t)$ are averaged out in (46).

[36] If z_t equals the lowest productivity state, z_t^- does not exist. In this case, $RI_{t,X}$ is defined as $RI_{t,X} \equiv \mathcal{I}_X^+(z_t,\mu_t)$. Likewise, $RI_{t,X} \equiv -\mathcal{I}_X^-(z_t,\mu_t)$ if z_t equals the highest productivity state.

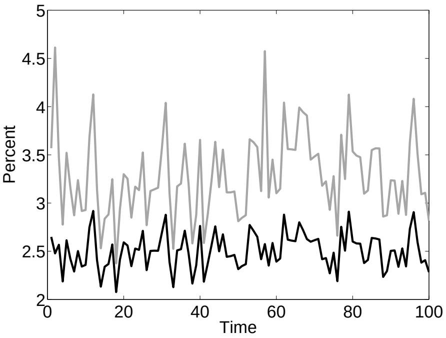
Figures 13 and 14 depict the responsiveness index for the last 100 periods of the simulation described above. The impulse responses to technology shocks upon impact are time-varying. Panels (a) to (d) in figure 13 show that the initial response of the target capital level, investment, output, and employment is not only larger on average but also more volatile in this paper's VULIM compared to the SLIM. However, this pattern does not exclusively pertain to the lumpy investment models. As shown in panels (e) to (h) in figure 13, a similar change in the responsiveness to technology shocks results when utilization is allowed to vary in a frictionless model.

However, the pattern is different for consumption. With variable utilization, the initial response of consumption becomes more stable in the lumpy model but more volatile in the frictionless model (cf. panels (a) and (c) of figure 14). The figure also shows that the response of utilization is smaller and less volatile in the VULIM compared to the VUFM (panel (b)). Finally, panel (d) of figure 14 reveals that the fraction of adjusting firms is more responsive to technology shocks in the VULIM compared to the SLIM, and the responsiveness is more volatile.

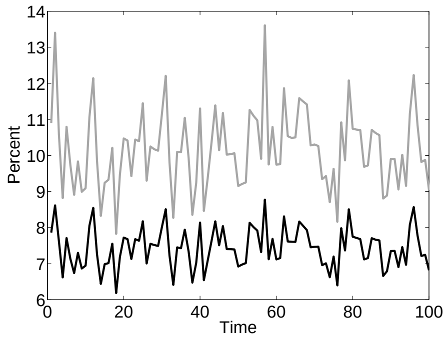
Overall, the analysis of the responsiveness index is in line with the above discussion on the impulse responses in figure 12. On the one hand, the variability of utilization affects not only the average response upon impact but also its variation over the business cycle. On the other hand, the impact of variable utilization is similar in the lumpy and the frictionless model. Thus, there is no evidence for substantial macroeconomic effects of the magnified investment lumpiness due to reserve capacity building.

6 Conclusion

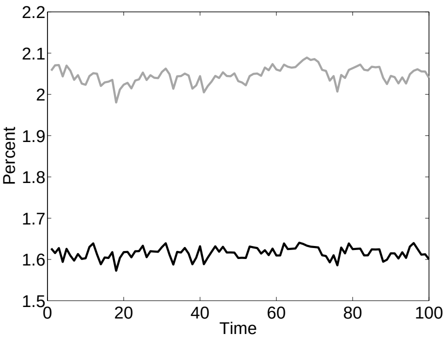
The lumpiness of investment at the plant level is well established. The macroeconomic relevance of lumpy investment, however, has been subject to debate. Previous lumpy investment models have assumed that firms constantly utilize their capital. However, this assumption is delicate because it attenuates the core incentive for lumpy investment, which consists in firms trying to avoid frequent fixed cost payments by investing more rarely, but larger amounts. Constant utilization makes investments which exceed the frictionless ideal increasingly costly because firms will have to fully utilize their new, "too large" capital stock. In contrast, with variable utilization, firms can undertake large investments to save on fixed cost payments



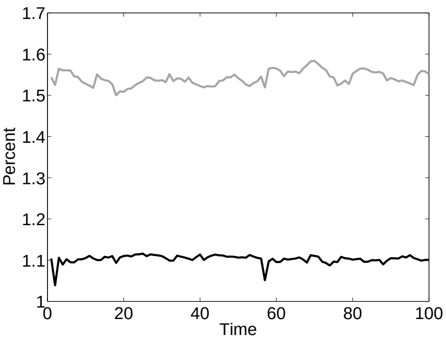
(a) Target capital, lumpy models



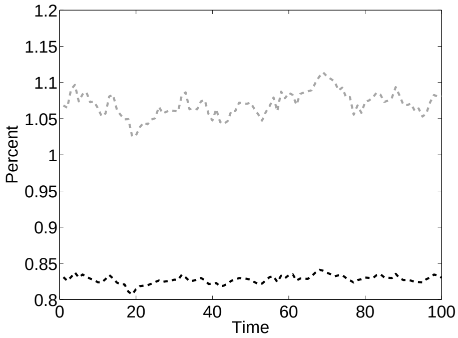
(b) Investment, lumpy models



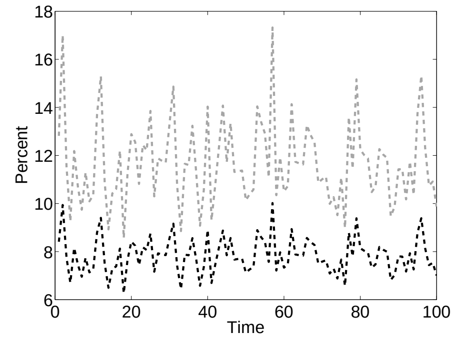
(c) Output, lumpy models



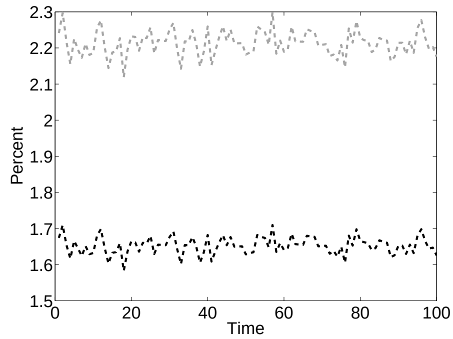
(d) Employment, lumpy models



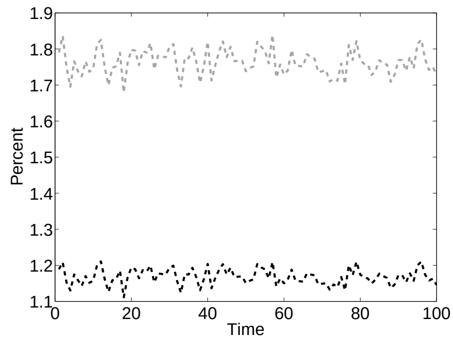
(e) Target capital, frictionless models



(f) Investment, frictionless models

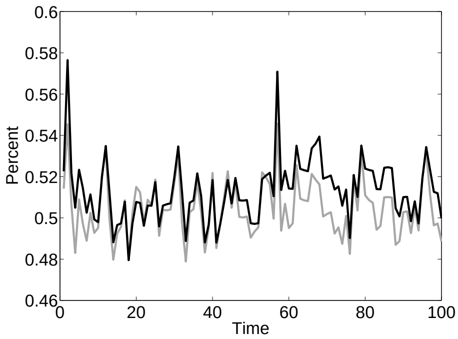


(g) Output, frictionless models

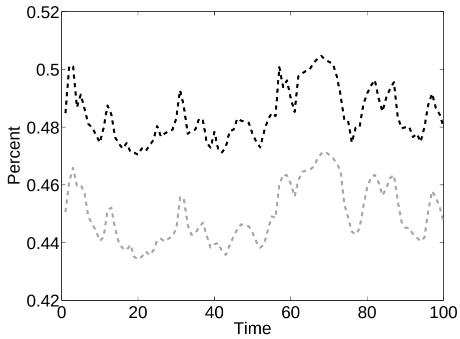


(h) Employment, frictionless models

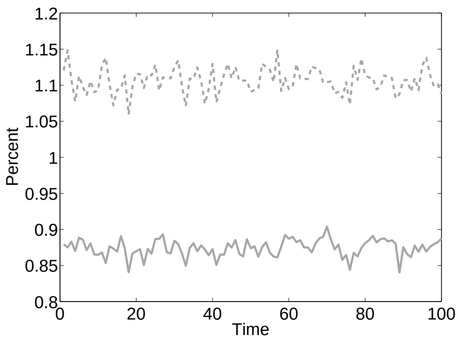
Figure 13: Responsiveness index $RI_{t,X}$ for different variables X: Target capital, investment, output, and employment. The index is depicted for various models: VULIM (gray solid lines), SLIM (black solid lines), VUFM (gray dashed lines), and SFM (black dashed lines).



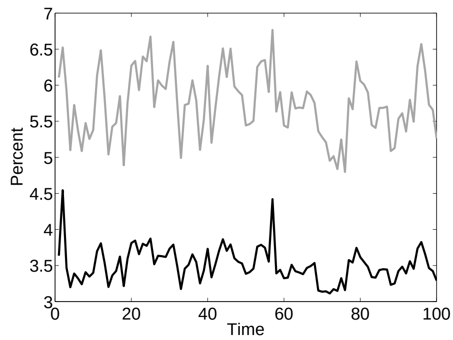
(a) Consumption, lumpy models



(c) Consumption, frictionless models



(b) Utilization



(d) Fraction of adjusting firms

Figure 14: Responsiveness index $RI_{t,X}$ for different variables X: Consumption, utilization, and the fraction of adjusting firms. The index is depicted for various models: VULIM (gray solid lines), SLIM (black solid lines), VUFM (gray dashed lines), and SFM (black dashed line).

while at the same time achieve the optimal amount of capital services by adjusting utilization accordingly. Thus, investment and utilization decisions are inherently interrelated. Firm-level investment lumpiness is enhanced by the possibility to build up reserve capacity that is not necessarily entirely utilized in the short run.

This paper empirically demonstrates the importance of capacity utilization for firms' investment decisions. Utilization has a positive effect on the probability of investment and a negative impact on the probability of disinvestment. Moreover, there are significant interaction effects with GDP growth. Thus, the rate of capacity utilization also influences how the investment probability responds to GDP. An aggregation exercise suggests that if utilization is restricted to be constant, the fraction of investing firms is underpredicted in booms and overpredicted in recessions.

Regarding the theoretical contribution, this paper is the first to analyze (i) optimal firm-level decisions in an environment with fixed capital adjustment costs and variable utilization and (ii) the macroeconomic consequences thereof. To this end, I extend a lumpy investment dynamic stochastic general equilibrium model by variable utilization. The model features fixed costs of capital adjustment, firm heterogeneity in capital, and aggregate productivity shocks. The firms' decision problem is complex: They need to make a discrete decision on investment and to choose labor, utilization, and next period's capital stock (if they invest). The utilization decision is intratemporal for investing firms and intertemporal for non-investing firms. Thus, firms not paying the fixed capital adjustment costs still have a limited intertemporal choice, a feature whose absence in many previous studies has been criticized. The model is solved using numerical methods.

The findings show that variable utilization manyfold affects firms' optimal decision rules. First, the results provide evidence for amplified lumpiness: While fewer firms invest on average, the adjusting firms invest a considerably larger amount. The increase in the kurtosis of firm-level investment rates is also indicative of enhanced lumpiness in the variable utilization model. Second, the findings suggest that reserve capacity building is important: Firms with a large capital stock choose a utilization rate substantially below the feasible maximum. Third, variable utilization affects firms' decisions on labor and utilization. Compared to the standard lumpy investment model, high capital firms feature smaller utilization and labor demand whereas low capital firms rely more extensively on these two production factors. Fourth, variability of utilization alters the cyclical properties of firms' optimal decisions. Both the target capital level of investing firms and the fraction of those firms fluctuates to a greater extent across the cycle when utilization is variable. Finally, the findings indicate that non-investing firms tend to choose a considerably lower utilization rate than investing firms.

Four models are compared to assess the macroeconomic implications of lumpy investment when utilization is variable: models with or without variable utilization and with or without fixed capital adjustment costs. Simulation results suggest that the fraction of adjusting firms is lower and that aggregate capital, the investment of adjusting firms and the target capital level are larger in this paper's VULIM. For the other macroeconomic quantities, the means are similar across the four models. The differences are more pronounced for standard deviations and correlations. The crucial question, however, is whether a model featuring both lumpy investment and variable utilization shows properties beyond what could be expected from the models with only one of those features. After all, the firm-level evidence has revealed an additional channel for lumpiness through reserve capacity building. However, the standard deviations and correlations of aggregate quantities change in a similar way when variable utilization is introduced in a lumpy or in a frictionless model.

A similar finding holds for the impulse response functions to aggregate technology shocks. The initial responses of output, consumption, investment, and employment are similar in this paper's VULIM and the VUFM. Pronounced differences pertain to the initial response of aggregate utilization (which is smaller in the VULIM) and the responses of aggregate capital, investing firms' target capital and the fraction of adjusting firms (which are larger in the VULIM).

Several reasons may explain why the amplified lumpiness at the firm level does not translate more clearly to the macroeconomic level. Of course, aggregation and general equilibrium price movements act as a smoothing device. In addition, there is a reason specifically pertaining to the variable utilization model: While large investments in reserve capacity lead to a jump in (firm-level) capital, they do not cause a correspondingly large increase in output and labor demand because part of the capital is left idle. Thus, even at the firm-level, the magnified lumpiness of investment and capital does not necessarily spread to capital services. Hence, while reserve capacity building enhances lumpiness at the firm level, it may at the same time serve as a potential explanation for the small macroeconomic impact of this magnified lumpiness.

Overall, the findings of this paper are broadly in line with the previous literature. I find a somewhat larger macroeconomic role of investment lumpiness than Thomas (2002) and Khan and Thomas (2003, 2008), possibly because the fixed capital adjustment costs are substantially larger in my model. Yet, my finding that, despite the larger fixed costs, the amplified lumpiness is of minor macroeconomic relevance is consistent with their results.

This study complements previous research on macroeconomic effects of lumpy investment. It extends the basic lumpy investment model by variable utilization while other studies have for instance considered additional aggregate shocks or firm heterogeneity in productivity. Combining such model properties with variable utilization would be a straightforward extension of this paper, although computational possibilities might still restrict the complexity of the model to be solvable. Computational feasibility also limits the number of state variables that can be used to replace cross-sectional distributions in the state vector, a limitation which this paper shares with other studies. Finally, this paper has not re-calibrated the lumpy investment model including variable utilization, but relied mostly on the calibration of Bachmann and Bayer (2014) due to the lack of a better dataset. In particular, I am not aware of an extensive firm-level dataset including quantitative data on both investment and utilization. Thus, further data collection would be valuable to improve both the calibration of the theoretical model and the empirical analysis of firms' interrelated decisions on utilization and investment.

In summary, this study has taken a step in the direction of understanding the role of reserve capacity for lumpy investment and its macroeconomic consequences. Building up reserve capacity provides both an explanation for enhanced lumpiness at the microeconomic level and a potential reason for its low macroeconomic relevance: Even if the adjustment of capital is lumpy due to fixed costs, the firm-level usage of this production factor may in fact be smoother, thereby attenuating the macroeconomic impact of lumpiness.

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A Figures

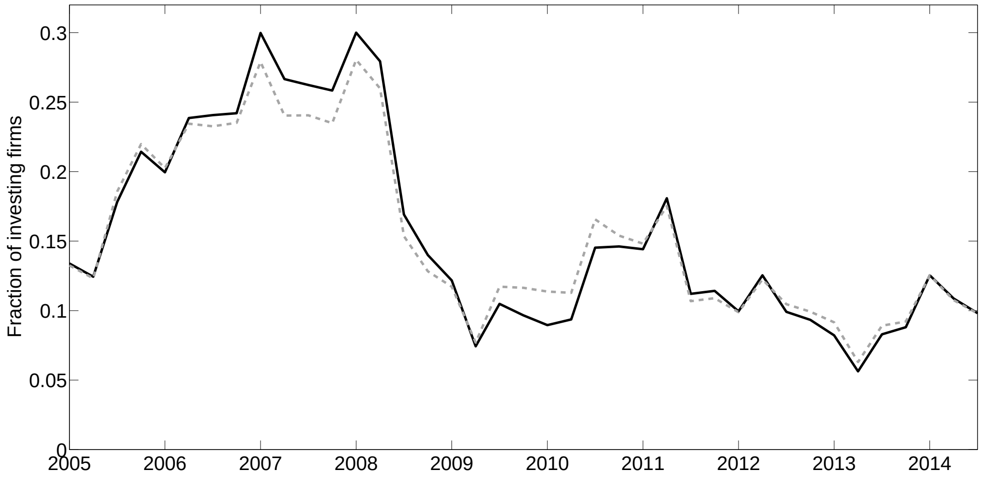

Figure 15: The fraction of investing firms when capacity utilization is fixed (gray dashed line) or variable (black solid line). The estimation is based on column (2) in table 2.

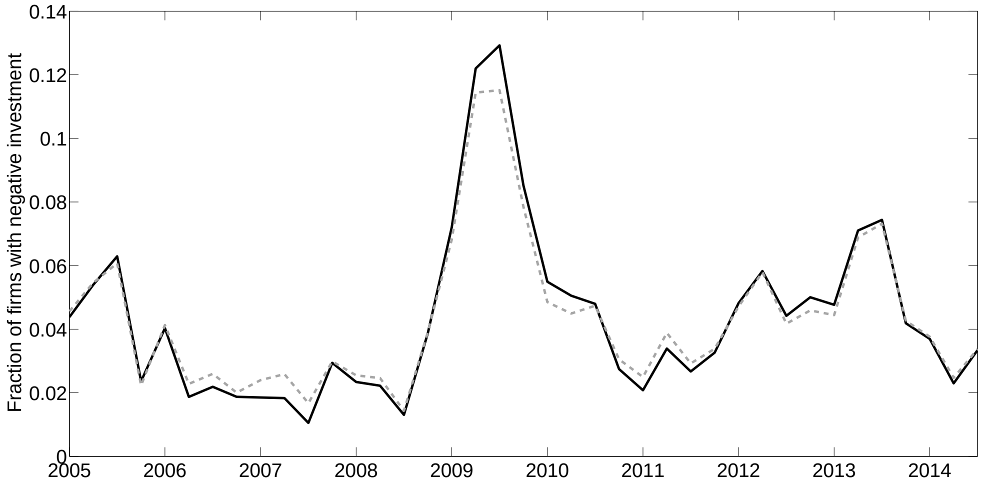

Figure 16: The fraction of firms with negative investment when capacity utilization is fixed (gray dashed line) or variable (black solid line). The estimation is based on column (2) in table 3.

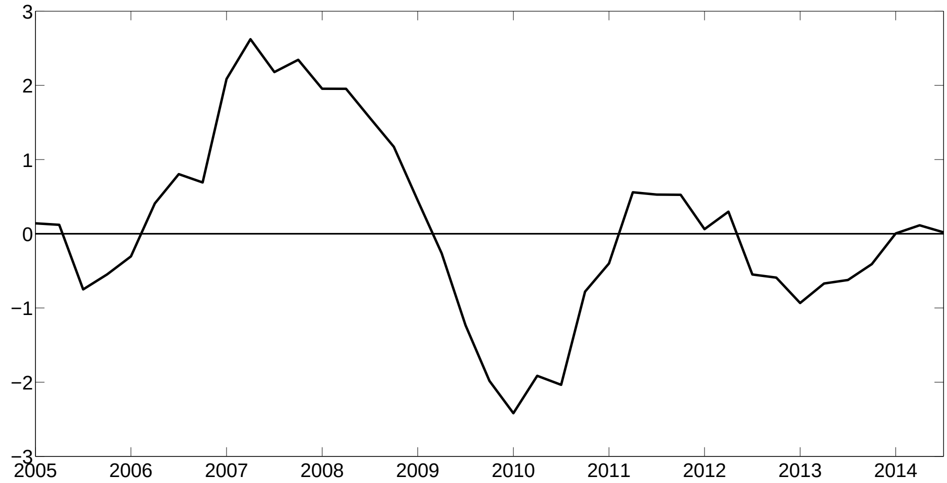

Figure 17: Difference in the predicted fraction of investing firms when utilization is variable or fixed. The difference is based on the FE model (column (2) in table 2).

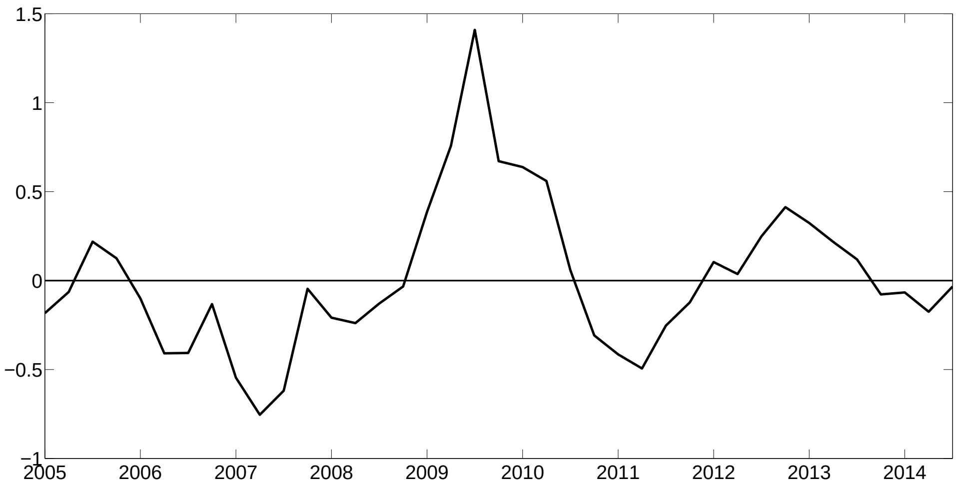

Figure 18: Difference in the predicted fraction of firms with negative investment when utilization is variable or fixed. The difference is based on the FE model (column (2) in table 3).

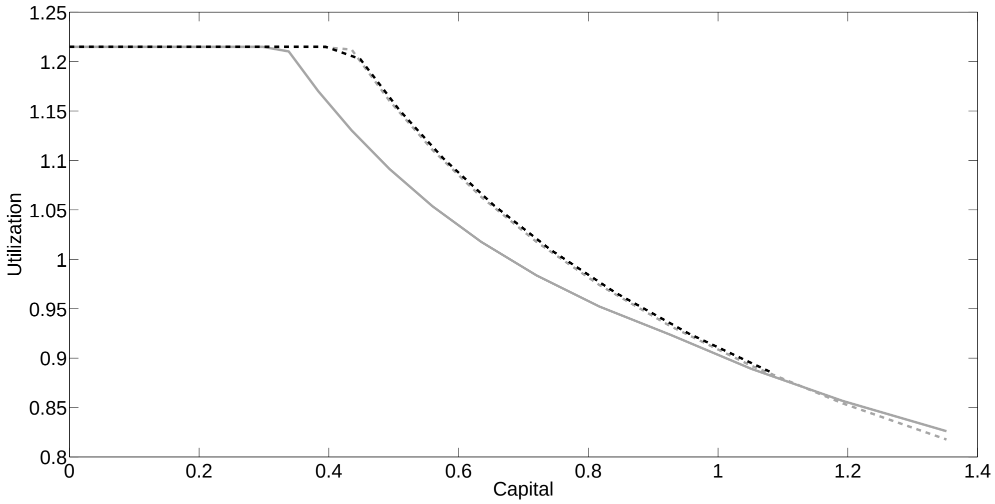

Figure 19: Utilization of the VULIM (gray lines) and the VUFM (black line). Dashed lines indicate investing firms, the solid line non-investing firms. In the VUFM, all firms invest.

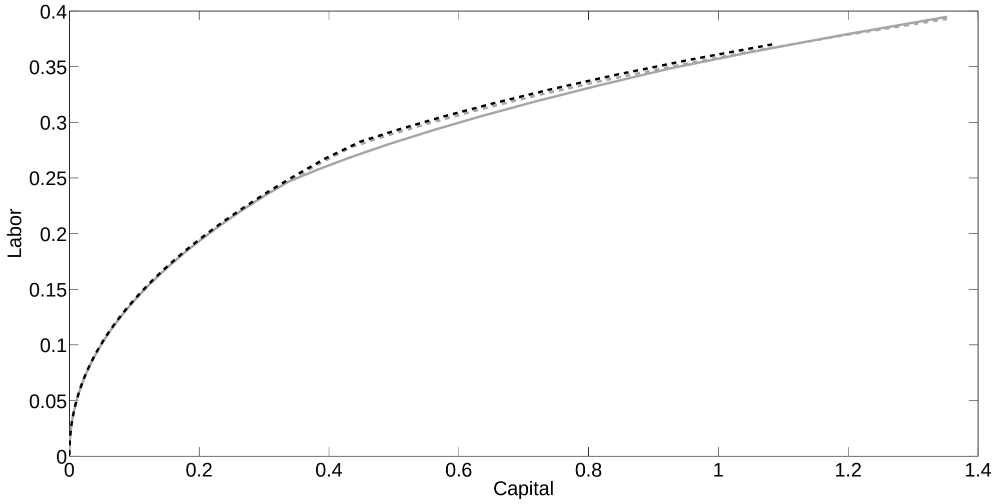

Figure 20: Labor demand of the VULIM (gray lines) and the VUFM (black line). Dashed lines indicate investing firms, the solid line non-investing firms. In the VUFM, all firms invest.

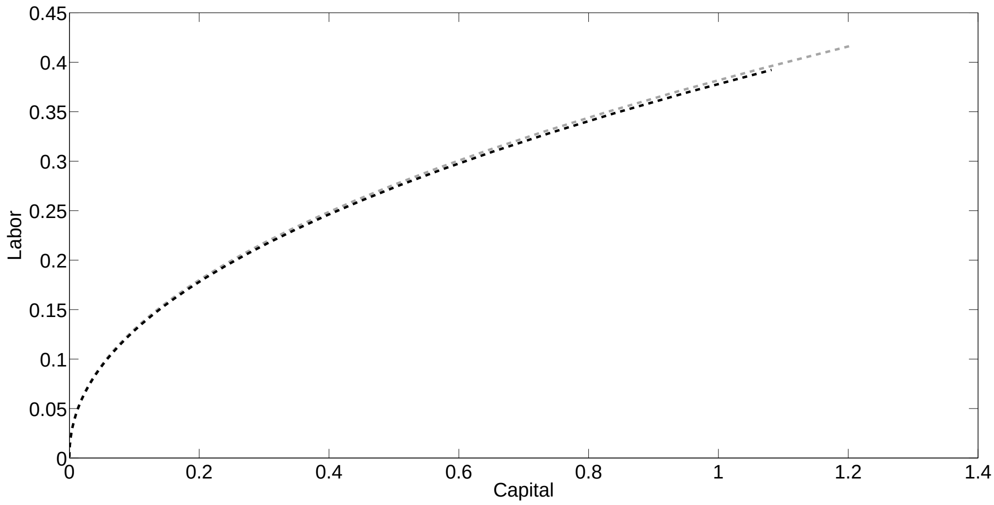

Figure 21: Labor demand of the SLIM (gray line) and the SFM (black line). In the SLIM, the labor demand of investing and non-investing firms is identical. In the SFM, all firms invest.

B Tables

Table 12: Target capital as a function of aggregate productivity

Productivity	VULIM	SLIM	VUFM	SFM
$z = 0.93$	1.02	0.98	0.69	0.70
$z = 0.95$	1.10	1.00	0.70	0.71
$z = 0.97$	1.16	1.04	0.71	0.72
$z = 0.98$	1.23	1.09	0.73	0.73
$z = 1.00$	1.28	1.13	0.74	0.74
$z = 1.02$	1.34	1.17	0.75	0.75
$z = 1.03$	1.39	1.21	0.77	0.76
$z = 1.05$	1.44	1.24	0.78	0.77
$z = 1.07$	1.48	1.28	0.80	0.79

Table 13: Mean investment rate as a function of aggregate productivity

Productivity	VULIM	SLIM	VUFM	SFM
$z = 0.93$	0.16	0.16	0.08	0.09
$z = 0.95$	0.18	0.17	0.09	0.10
$z = 0.97$	0.18	0.17	0.10	0.10
$z = 0.98$	0.19	0.18	0.10	0.10
$z = 1.00$	0.20	0.18	0.11	0.11
$z = 1.02$	0.20	0.19	0.11	0.11
$z = 1.03$	0.21	0.19	0.12	0.12
$z = 1.05$	0.22	0.20	0.12	0.12
$z = 1.07$	0.22	0.20	0.13	0.12

Notes: Table entries are computed using the simulation described in section 5.2.4.

C Uniqueness of Goods and Labor Market Equilibrium

The goods market equilibrium (32) requires that consumption plus investment equals output. If production of the good is increasing and total demand for the good is decreasing in p, there can be at most one equilibrium.[37] Totally differentiating $p = U_1(C, 1 - N^h)$ and rewriting yields:

$$\frac{dC}{dp} = \frac{1}{U_{11}(C, 1 - N^h)} + \frac{U_{12}(C, 1 - N^h)}{U_{11}(C, 1 - N^h)} \frac{dN^h}{dp}. \tag{49}$$

In the lumpy investment literature, utility is usually assumed to be additively separable in consumption and leisure (i.e., $U_{12} = 0$). In this case, $dC/dp < 0$ immediately follows from decreasing marginal utility of consumption ($U_{11} < 0$).

[37] More precisely, I need either production to be strictly increasing or demand to be strictly decreasing in p while the other curve can be (weakly) decreasing or increasing.

The analysis of aggregate investment and output's dependence on p is more demanding. Investment is decreasing in p if the following condition holds:[38]

$$\int_{\mathcal{K}} \left[\left(\gamma \frac{\partial k^*}{\partial p} + \delta_u \frac{\partial u_I}{\partial p} k \right) G\left(\bar{\kappa}\right) + \left(\gamma k^* - \left(1 - \delta(u_I)\right) k\right) g\left(\bar{\kappa}\right) \frac{\partial \bar{\kappa}}{\partial p} \right] \mu(dk) \leq 0, \tag{50}$$

where $g\left(\kappa\right) = \partial G\left(\kappa\right) / \partial \kappa$. This expression captures both the intensive and extensive margin of investment. The size of each firm's capital adjustment depends on $\partial k^* / \partial p$ and $\partial u_I / \partial p$ while $g\left(\bar{\kappa}\right) \partial \bar{\kappa} / \partial p$ determines the change in the fraction of firms undertaking capital adjustments.

For aggregate output to be increasing in p, the following condition needs to hold:

$$\int_{\mathcal{K}} \left[\frac{\partial y_N}{\partial p} \left(1 - G\left(\bar{\kappa}\right)\right) + \frac{\partial y_I}{\partial p} G\left(\bar{\kappa}\right) + \left(y_I - y_N\right) g\left(\bar{\kappa}\right) \frac{\partial \bar{\kappa}}{\partial p} \right] \mu(dk) \geq 0, \tag{51}$$

where $y_N = zF\left(ku_N, n_N^f\right)$ and $y_I = zF\left(ku_I, n_I^f\right)$. Thus, it is not sufficient to show that both investing and non-investing firms increase their production when p rises. In addition, one needs to consider that firms may switch from investing to non-investing and vice versa, which involves a jump in production. This is captured by the term $\left(y_I - y_N\right) g\left(\bar{\kappa}\right) \partial \bar{\kappa} / \partial p$.

Conditional on p, which is determined in the goods market, I analyze the uniqueness of the labor market equilibrium. I present conditions under which labor supply N^h is non-decreasing and aggregate labor demand is decreasing in the real wage w, in which case there is at most one equilibrium in the labor market.

Totally differentiating $w = U_2(C, 1 - N^h)/p$ and rewriting yields:

$$\frac{dN^h}{dw} = \frac{U_{21}(C, 1 - N^h)}{U_{22}(C, 1 - N^h)} \frac{dC}{dw} - \frac{p}{U_{22}(C, 1 - N^h)}. \tag{52}$$

Assuming additive separability (i.e., $U_{21} = 0$), $U_{22} \leq 0$ implies that labor supply is non-deceasing in w.

For aggregate labor demand to be decreasing in w, the following condition needs to hold:

$$\begin{aligned} \int_{\mathcal{K}} \Bigg[\frac{\partial n_N^f}{\partial w} \left(1 - G\left(\bar{\kappa}\right)\right) + \frac{\partial n_I^f}{\partial w} G\left(\bar{\kappa}\right) + \left(n_I^f - n_N^f\right) g\left(\bar{\kappa}\right) \frac{\partial \bar{\kappa}}{\partial w} \\ + \bar{\kappa} G(d\bar{\kappa}) \frac{\partial \bar{\kappa}}{\partial w} \Bigg] \mu(dk) < 0. \end{aligned} \tag{53}$$

[38] The dependencies on $(k; z, \mu)$ are suppressed for readability.

$\partial n_N^f / \partial w < 0$ and $\partial n_I^f / \partial w < 0$ follows from (22), (23), (25), (26), and the assumptions $F_{11} < 0$, $F_{22} < 0$ and $F_{12} \geq 0$. However, to assure that (53) is satisfied, one needs to additionally consider the difference in the labor demand of firms switching their binary investment decision, which is captured by the term $\left(n_I^f - n_N^f\right) g\left(\bar{\kappa}\right) \partial \bar{\kappa} / \partial w$, and the change in aggregate fixed costs (which are denominated in hours of labor) resulting from firms which change their capital adjustment decision.

In summary, if household utility is additively separable and if conditions (50), (51) and (53) hold, then there is at most one equilibrium in the goods and labor market. A proof of these conditions, however, is beyond the scope of this paper. Nevertheless, I can present numerical evidence for uniqueness of the goods and labor market equilibrium for this paper's model specification and calibration.

Figure 22 shows aggregate demand (consisting of consumption and investment) and aggregate production in the steady state of this paper's model. The equilibrium is clearly unique as demand is strictly decreasing and output strictly increasing. A more detailed analysis reveals that both consumption and investment are strictly decreasing, the latter being far from linear. The desired capital stock k^* also decreases in p, while the fraction of firms adjusting their capital stock first declines and then rises. Figure 23 shows the labor market equilibrium conditional on the equilibrium p. As for the goods market, there is no evidence of multiplicity of equilibria.

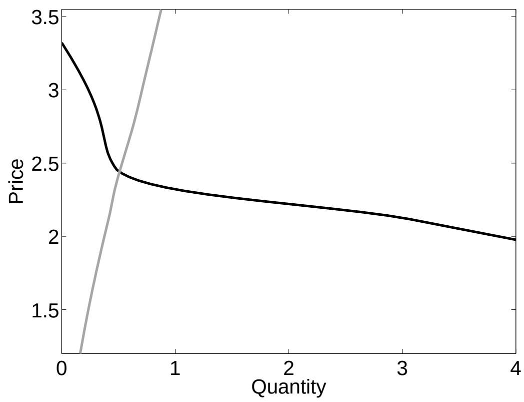

Figure 22: Consumption and investment demand (black line) and production (gray line) as a function of p in the steady state.

Uniqueness does not only hold in the steady state. A simulation of the model over 5200 periods, starting from the steady state, reveals a unique equilibrium in the goods and labor market in each period. In particular, both consumption and aggregate investment are strictly

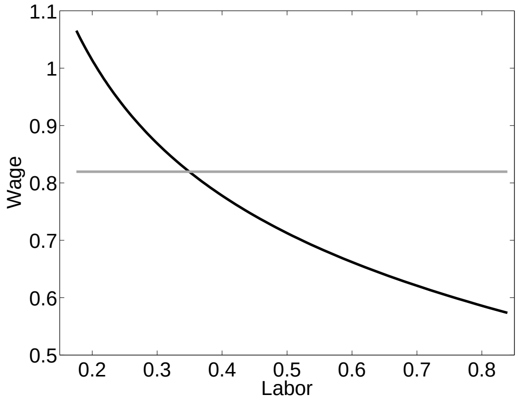

Figure 23: Labor demand (black line) and supply (gray line) as a function of w (conditional on equilibrium p) in the steady state.

decreasing in p, aggregate output is strictly increasing in p, and aggregate labor demand is strictly declining in w (for a given p).[39] Thus, for the model specification and calibration in this paper, multiplicity of equilibria should not be an issue.

[39]Labor supply is perfectly elastic for the specification of household utility used in this paper. Thus, labor supply determines w conditional on p, and the strictly decreasing labor demand uniquely determines the aggregate amount of labor.